DRAG FACTOR AND COEFFICIENT OF FRICTION IN TRAFFIC ACCIDENT RECONSTRUCTION

Topic 862 of the Traffic Accident Investigation Manual

by

Lynn B. Fricke
and
J. Stannard Baker

1. Introduction ....................................................... 3
2. Definition of Friction ............................................ 3
3. Basic Assumptions for Friction ................................. 4
4. Definition of Drag Factor ....................................... 7
5. Friction Circle .................................................... 8
6. Drag Factor/Friction Coefficient for an Actual Accident .......... 9
7. Velocity Equations for Skidding .............................. 13
8. Speed Estimates from Irregular Skidmarks ...................... 20
9. Drag Factors After a Collision .................................. 28
10. Vehicle Center of Mass Location ............................... 35
11. Other Acceleration Values ..................................... 37
12. Summary .......................................................... 37
13. Sources .......................................................... 38

NORTHEASTERN UNIVERSITY TRAFFIC INSTITUTE
DRAG FACTOR AND COEFFICIENT OF FRICTION IN TRAFFIC ACCIDENT RECONSTRUCTION

1. INTRODUCTION

The rate at which vehicles decelerate from braking is often of great concern in traffic accident reconstruction. If the distance the vehicle decelerated is known, along with its rate of deceleration, the vehicle's initial velocity can often be calculated using the equation

\[ v_i = \sqrt{v_e^2 - 2ad} \]  

where

- \( v_i \) = initial vehicle velocity in ft/sec or m/sec
- \( v_e \) = end vehicle velocity in ft/sec or m/sec
- \( a \) = the vehicle's acceleration in ft/sec/sec or m/sec/sec (\( a \) is negative, \(-\), when the vehicle is slowing)
- \( d \) = distance the vehicle accelerated (or decelerated) in ft or m.

(This equation is discussed in some detail in Topic 860; see that topic if you are unfamiliar with its use.) The most difficult problem is usually encountered when the acceleration/deceleration of the vehicle must be determined. A vehicle's acceleration or deceleration is related to its drag factor, \( f \), and the coefficient of friction, \( \mu \) (Greek letter mu). Drag factor is related to acceleration by the following equation:

\[ a = fg \]  

where

- \( f \) = drag factor (no dimensions)
- \( g \) = the acceleration of gravity (either 32.2 ft/sec/sec or 9.81 m/sec/sec).

Drag factors and coefficients of friction are generally similar for all types of four-tired vehicles (essentially, passenger cars and pickup trucks). However, drag factors for motorcycles and heavy vehicles (large trucks, both single-unit and articulated) can often differ from those for four-tired vehicles. This topic deals primarily with the latter. For a discussion of motorcycles and heavy vehicles and their drag factors, consult Topics 874 and 878.

2. DEFINITION OF FRICTION

Friction can be thought of as the resisting force to motion between two surfaces at their interface (contact). We are accustomed to thinking of this as a bad thing. After all, considerable money is spent every year in attempting to reduce friction in engines. When it comes to stopping, however, friction from brakes is a very good thing.

A more precise definition of the coefficient of friction is the ratio of the tangential force (parallel to the surface) applied to an object sliding across a surface to the normal force (perpendicular to the surface) on the object. A typical diagram you will see in an engineering mechanics book is shown in Exhibit 1. It might be easier to visualize what is going on if the object is sliding on a level surface as shown in Exhibit 2. In this simpler diagram the coefficient of friction, \( \mu \), is simply the horizontal force divided by the vertical force which is the object's weight when the object is sliding across the surface.

\[ \mu = F/w \]  

Generally, only three types of friction are considered in traffic accident reconstruction:

1. Static friction is defined using equation (3) (for a level surface) when sliding is just beginning. When an object is just beginning to slide, more force is required to start its movement than after movement has started. Thus, on a level surface the horizontal force, \( F \), is greater at the beginning of sliding than it is after movement has started. The force required to start the sliding is used in computing static friction.
Exhibit 1. The weight component pushing perpendicular to the surface is $w \sin \theta$ and the weight component parallel to the surface is $w \cos \theta$ as shown in the diagram.

2. Dynamic friction also uses equation (3) for a level surface. In this case, however, $F$ represents the force being applied after the object is sliding. This force is less than the horizontal force used in the static friction computation.

3. Rolling friction (rolling resistance) refers to the resisting forces that come into play when a vehicle is rolling with no braking. Generally these values are very low and are assumed to be insignificant in most accident reconstruction problems. Rolling friction is, however, important in tire design and other vehicle design considerations.

In hard braking (with non-antilock brakes) the time delay between the beginning of braking and full lockup of the first wheel is generally very short. Hard braking tests done using a Ford LTD at 40 mi/hr on a tar and gravel-chip pavement gave a delay of 0.12 sec for the first wheel to lock from the time the brakes were first applied. Reed suggests that wheel lockup could take slightly longer (in the neighborhood of 0.5 secs). Nevertheless, the amount of time that static friction could be considered is very short as compared to the total time a vehicle is normally skidding. Thus, for accident reconstruction purposes, there is little reason to be concerned with static friction. If you have a case with a vehicle equipped with air brakes, there may be a much greater delay in wheel lockup from the time brakes were first applied. For a more thorough discussion of heavy-truck braking, see Topic 878.

3. BASIC ASSUMPTIONS FOR FRICTION

For sliding friction, several conditions are generally always expected. With a rubber tire sliding on pavement, however, some of these conditions might not be present. The general properties of friction are as follows:

1. If the sliding surface is horizontal, the horizontal force required to slide the object is proportional to the object's weight. Thus, if the weight of the sliding object is increased by 20 percent, then the required horizontal force to cause it to slide is also increased by 20 percent.

2. Dynamic friction is lower than static friction. That is, after an object starts sliding, it takes less force to keep it sliding.

3. The friction force does not depend on how much area of the sliding object is in contact with the surface. That is, if you increased the area that is in contact with the surface over which an object is sliding (keeping everything else constant), the amount of force required to slide the object would stay the same.

4. The friction force does not change when velocity changes. Thus, the friction force at a higher sliding speed would be the same as that at a lower sliding speed.

5. The friction force does not change when the temperature changes. That is, the friction force is the same for an object sliding over a surface at 80°Fahrenheit as it is at 20°Fahrenheit.

Friction Force Proportional to Weight

If you are on a level surface and sliding a car with locked wheels, there is an increase in the horizontal friction force if weight is added to the car. If the weight is increased by 20 percent, you can expect the horizontal friction force to be increased by approximately 20 percent. Generally, it is expected that friction capability will decrease some with increased load.

If tires on a large, heavy truck are tested for friction coefficient over a given pavement and are compared to tire friction values on a much smaller passenger car, do not expect to see the same friction values. This is because of the far greater load placed on truck tires and other inherent differences between truck tires and passenger car tires.

Exhibit 2. For a level surface the coefficient of friction is simply the ratio of the horizontal force required to slide an object to the weight of the object.
For passenger cars the friction force (with locked wheels and similar tires) that can be generated over a given pavement does not differ significantly between sizes of cars. For example, if you have a compact car and an intermediate/full-size car sliding with locked wheels at 30 mi/hr over the same pavement with similar tires, expect to get essentially the same friction values.

**Dynamic and Static Friction**

In discussing the transition from static to dynamic friction, it is useful to consider how the drag of the road on a tire varies from onset to a locked wheel skid. This is illustrated in Exhibit 3. Before braking, the wheel has full rotation; but where the tire touches the road surface, there is, momentarily, no movement relative to the road surface. Compared to the vehicle's speed, the speed of the tire in contact with the road is then 0 percent. As soon as brakes are applied, a retarding force on the tire is developed by the road. When this force begins, elastic tire material is pressed into road surface irregularities (asperities). Thus, the drag force stretches the tread rubber for an instant while the body of the tire begins to slow down. At this point, the speed of the tire close to the road surface compared to the vehicle's speed has increased.

As the braking force is increased, the tire loses its grip on the pavement and starts to slip. As slipping increases, the amount of time for the rubber to grip the road surface becomes less and less. When the wheel reaches lockup, the tire now has 100 percent slip. At 100 percent slip, the tire is sliding with dynamic friction. As can be seen in Exhibit 3 (which is for a particular tire), the friction force reaches a maximum at 10 to 20 percent slip for the tire.

For panic braking situations, drivers generally apply their brakes very rapidly and very hard. This will lock the wheels rapidly (for non-antilock brakes), thus causing nearly all of the braking to occur at 100 percent slip.

Exhibit 3. As a tire first starts to slip on a hard surface, the friction force increases and reaches a maximum. Then the friction force decreases somewhat. There is not much difference in the friction force at high values of slip.
Friction Force Related to Sliding Area

In some instances a greater tire-pavement interface area will contribute to a higher friction force. For bald (no tread grooves) tires on a clean, dry, hard surface, it is possible that a higher sliding force will be generated\textsuperscript{2}. Definitely, a lower value would not be expected. Also, a higher value of friction can be obtained with bald tires on ice.

For wet pavement conditions, bald tires do not give better friction values. Tires are designed with grooves and sipes (see Topic 825) which allow a place for the water to go when the tire contacts the wet pavement. Thus, as a pavement becomes wet the presence of tread becomes increasingly important.

Velocity Effects on Friction

The effects of velocity on friction on typical passenger-car tires is noticeable. For years, the table given in the Traffic Accident Investigation Manual has suggested that higher values are expected at speeds lower than 30 mi/hr for the same type of pavement. Research in the area supports this. In Exhibit 4 Ohio researchers\textsuperscript{1} took average friction values for four vehicles tested over three surfaces at the indicated speeds. Small, intermediate and large cars were represented. A pickup truck was also tested. A conclusion of their study was:

\ldots the average calculated friction coefficient decreases as speed increases, falling rapidly as the nominal prebraking speed goes from 10 to 30 mi/hr and slowly between 30 and 60 mi/hr.

This indicates that if tests are done with a vehicle to determine the coefficient of friction, it would be appropriate to do them above or below 30 mi/hr — whichever is closer to the accident vehicle's speed. Collins\textsuperscript{4} suggests reduction fac-

---

\textit{Exhibit 4.} It is generally found with normal passenger car tires that the coefficient of friction reduces with increasing speed. This exhibit shows the results of one such study.
tors for coefficients of friction where the expected accident speed is greater than 40 mi/hr. Friction reduction factors are used in some computer programs.

Temperature Effects on Friction

The effects of temperature on the coefficient of friction are insignificant unless radical temperature differences are encountered. In general, friction will decrease slightly with an increase in air temperature. Reference 2 indicates that over a temperature range of 45° to 80° Fahrenheit on a dry asphalt surface, friction only decreased 0.10. On a wet surfacethe difference was even less. Compared to other factors, then, temperature effects are so minimal as to be rarely worth considering. If you have reason to be concerned about them, just be sure to do your friction test on a day with a similar temperature.

Other Considerations for Friction

Snow tires have little effect on braking with locked wheels over a snow-packed surface. Studded tires can enhance braking over icy surfaces, but can reduce friction under other conditions. The effect of tire inflation pressure is sometimes raised. A higher air pressure than normal for a given tire reduces the contact area of a tire, just as lower tire pressures tend to increase the contact area. The effect of this small difference will have insignificant effects in most cases. High-performance tires may give higher friction values than normal passenger tires. This usually does not cause problems when speed estimates are done. Simply use the lower, "normal" friction coefficient to obtain a conservative estimate or do tests with similar high-performance tires. Antilock braking systems (ABS) do not operate at 100 percent slip. Thus, the performance of ABS-equipped cars could be better than that of locked-wheel-braked cars. Other advantages of ABS-equipped cars will be discussed later.

4. DEFINITION OF DRAG FACTOR

The term drag factor will not be found in typical engineering mechanics or physics books. It has been used for many years in traffic accident investigation/reconstruction. Drag factor is given the symbol f. Drag factor is defined as the force required for acceleration (or deceleration) in the direction of the acceleration (or deceleration) divided by the object's (vehicle's) weight. In equation form drag factor is defined as:

\[ f = \frac{F}{w} \]  (4)

Often some confusion is created when equations (3) and (4) are compared. The right sides of both equations are clearly the same. The difference is that equation (3) deals with coefficient of friction, \( \mu \), and equation (4) deals with drag factor, \( f \). For \( \mu \), the object must be sliding across the surface. This is not the case for drag factor. Drag factor and coefficient of friction will be equal only in cases where all wheels are locked and sliding on a level surface.

Drag Factor Related to Vehicle Types

Consider a motorcycle on a level surface with the rear wheel locked and the front wheel rotating with no braking. There is sliding of the rear wheel on the pavement. Thus, the retarding force at the rear wheel is dependent on the coefficient of friction. However, not all wheels in contact with the road are braking. Therefore, the drag factor on the motorcycle is clearly not the same as the coefficient of friction between the rear wheel and the road surface. Actually, the drag factor of the motorcycle will be less than the coefficient of friction. A more detailed discussion of motorcycle braking is given in Topic 874.

Like motorcycles, tractor and semi-trailer vehicles can have different drag factors at each axle. The wheels on one axle may be locked, while wheels on another axle may have much less than 100 percent slip. Indeed, it is possible that the wheels on a particular axle may have different drag factors. Nevertheless, the drag factor on the whole vehicle is one value.

After a collision, the damaged vehicles usually travel some distance before they decelerate to a stop. Wheels are often jammed and will not rotate as a result of the collision. Also, the damaged vehicles are usually rotating as they move off to their rest positions. With a rotating vehicle the drag factor will most likely be changing as it rotates and moves to its rest position. In all cases there is a drag factor associated with the deceleration of the vehicle to a stop.

Drag Factor Related to Gravity

Earlier in this topic you were given the relationship between acceleration \( a \), drag factor \( f \), and the acceleration of gravity \( g \) in equation (2):

\[ a = fg \]  (2)

If you are using equation (1) to solve for the initial velocity when a vehicle is slowing, be sure to use a negative \( (−) \) value for acceleration \( a \). Drag factor \( f \) does not have a sign \( (+ \text{ or } −) \) associated with it. So if you use equation (2), you will need to add the negative sign. The use of this equation is discussed with sample problems in Topic 860.

Drag factor can easily be solved from equation (2) to get:

\[ f = \frac{a}{g} \]  (5)

So it can be seen that \( f \) is simply a ratio of acceleration of the object to the acceleration of gravity. Thus in free-fall, where acceleration, \( a \), is equal to the acceleration of gravity, \( g \), drag factor, \( f \), is equal to 1.0 \( (a = ag = g/g = 1.0) \).
Recall the basic definition of drag factor from equation (4):

\[ f = \frac{F}{w} \]

where \( F \) is the force in the direction of the acceleration and \( w \) is the weight of the object. The only time that drag factor could be equal to 1.0 is when the acceleration force, \( F \), is equal to the object's weight. So clearly when drag factor equals 1.0, the force on the object is equal to its weight and the acceleration is equal to \( g \).

Because drag factor is always a ratio of the acceleration of the object to the acceleration of gravity, drag factor may be thought of as the percentage of the acceleration of gravity expressed as a decimal. So if someone says, "That car decelerated at 0.5g," he means that the drag factor was equal to 0.5.

**Drag Factor for Increasing and Decreasing Velocity**

Usually, drag factor refers to conditions where a vehicle or object is slowing. Clearly, the word drag implies slowing. Nevertheless, drag factor or \( f \) can be used for cases where a vehicle or object has increasing velocity (positive acceleration) or decreasing velocity (negative acceleration). If you convert \( f \) to \( a \) (acceleration) to use in one of the equations given in the Manual, always be careful to use a positive value for \( a \) if the velocity is increasing and a negative value for \( a \) if the velocity is decreasing.

**5. FRICTION CIRCLE**

Essentially, all of the normally occurring forces on a car are transferred through the tires at their interface with the road surface. Other forces not considered are such things as air loads (wind resistance) and collision forces. If a car is steered, the wheels are turned and a side force is generated because of the slip angle between the plane of the tire and the direction of motion of the car. When a car is braked, the slip between tires and pavement might reach 100 percent. At that point there is no significant friction left to steer the car. For example, the skidmarks shown in Exhibit 5 were made by locked wheels due to hard braking. After the wheels were locked, the steering wheel was turned hard to the left. The car continued to travel straight ahead; the steering of the wheels had no appreciable effect. All of the available friction had been used in the 100 percent slip in braking. In Exhibit 6, which is a continuation of the tire marks shown in Exhibit 5, the brakes were released at Point A in the photograph. Then the car immediately started turning in the direction of the steering and ultimately went into a yaw.

If both of the rear wheels are locked on a passenger car while the front wheels continue to rotate, an unstable condition develops. Because the front wheels are free to rotate, they can continue to have side friction forces. However, with the rear wheels locked, they have no effective friction available to keep the rear of the car from sliding to the left or the
right. If the pavement has a cross slope, the rear tires will tend to slide in that direction. Once the rear has started to slide sideways with only the rear tires locked, the car will continue rotating (if there is enough speed) until the car is going backward. If the driver steers hard as the rear tires are locked, the back of the car will slide in the direction of the centrifugal force caused by the steering. An example of this phenomenon is shown in Exhibits 7 and 8. These marks were made by locking the rear wheels with the emergency brake. As in the case shown in Exhibits 5 and 6, all the friction was used in the braking (100 percent slip) and no other significant side force was available.

To describe more effectively what is happening in the two previous examples, the forces being applied to the tire will be discussed. This is often diagrammed as the friction circle shown in Exhibit 9. As you can see, the lateral and longitudinal directions of force are indicated on the tire. The longitudinal forces are either for braking or for driving the car forward. The lateral is, of course, for steering. If the tire could have equal friction both laterally and longitudinally, then the friction circle would indeed be a circle. Actually, the friction capability is not quite identical for every direction the force is applied to the tire, so the circle is more of an ellipse; but for this explanation a circle will be adequate. The maximum force available is the radius of the circle. The direction of the force indicates how much force is being used in the longitudinal and lateral directions.

In Exhibit 9A the force on the tire is a driving force and a steering force. The vector sum of the lateral and longitudinal force is clearly less than the maximum allowable. In Exhibit 9B there is some steering and a considerable amount of braking force. In Exhibit 9C the tire is skidding and the direction of motion is opposite the resultant friction force.

A more detailed discussion of the effects of braking and yawing together is included in the topic on yaws, Topic 872.

6. DRAG FACTOR/FRICTION COEFFICIENT FOR AN ACTUAL ACCIDENT

If you are working on an accident case in which you need to know the appropriate drag factor (friction coefficient) to use, there are several methods for determining this:

1. Test-skid the accident vehicle or an exemplar vehicle.
2. Slide a test tire (not the whole vehicle) to get a friction coefficient.
3. Use existing highway department skid numbers for the road in question.
4. Look up friction coefficients in a table and apply the appropriate adjustments to them for the case at hand.

Each method has its own merits and problems. The method you select will depend on such variables as time, cost, disruption to traffic, and safety.
**Test Skids Without Special Equipment**

For the ideal test, use the same vehicle that made the accident skidmarks (if it is not too severely damaged), at the same spot, with the same load and surface conditions. Drive the test vehicle at what you believe may have been the speed of the vehicle that made the marks. You need not make the tests at an extremely high speed just to be near the accident vehicle speed. If the ideal test situation is not possible — which is usually the case — then approximate it as closely as possible. It is important to use the same pavement in the same condition; it is less important to use the same vehicle. Drive the vehicle at the estimated accident speed. Again, do not go over a reasonably safe speed. A good test speed is the speed limit (if it is 35 mi/hr or less). Then, if the accident skids are longer than the speed-limit skids, there is direct proof that the vehicle's speed was greater than the speed limit. Hold the speed constant as the vehicle reaches the right place on the road, note the speed shown on the speedometer, hold the steering wheel steady, put on the brake very hard and very quickly, and hold it until the vehicle stops. Now measure the the length of each mark. Repeat the test at the same speed. If there is a difference of only 10 percent or less between the two tests (compare the longest skid from the first test to the longest skid from the second), consider the test adequate. If not, repeat the test until consistent results are obtained.

Good skid tests are more difficult to make than you might think. Adjusting speed and holding the indication on the speedometer can give trouble, but the main difficulty is applying brakes quickly and strongly. It may take several trials, especially with massive tires and without power brakes. If brakes are applied too slowly or not strongly enough, the vehicle loses considerable speed before wheels lock and tires slide. Skidmarks are then too short for the initial speed reading, and the calculated coefficient of friction too high, resulting in a high estimate of accident speed. Reed suggests that 15 to 30 percent of the initial energy of a car may be dissipated before clearly (easily) visible skidmarks are produced. (This does not mean that 15 to 30 percent of the velocity has been lost before the marks are seen). This clearly illustrates...
the importance of detecting where the vehicle first started to slow.

Speedometer error must be reckoned with. If there is any question about it, test the speedometer as follows. With the speedometer continuously indicating the same speed as when the test skids were made, measure how many seconds it takes to drive the measured mile between two mileposts (or the measured kilometer between two kilometer markers). Then divide 3,600 by the measured number of seconds to get the actual speed. For example, if it took 75 sec to cover a measured mile, the speed would be \( \frac{3,600}{75} = 48 \) mi/hr. If the speedometer was reading 50 mi/hr throughout this trial run, it would have indicated 2 mi/hr high. If the test skid had been made with the speedometer reading 50 mi/hr, the coefficient of friction calculation would have to be made on the basis of the actual speed of 48 mi/hr. Using the correct initial speed is extremely important. Say, for example, you did a test that resulted in 40 ft of skids. The drag factors you would get if the initial speed was 29, 30 and 31 mi/hr are shown in the table below.

<table>
<thead>
<tr>
<th>Speed (mi/hr)</th>
<th>Drag factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>.70</td>
</tr>
<tr>
<td>30</td>
<td>.75</td>
</tr>
<tr>
<td>31</td>
<td>.80</td>
</tr>
</tbody>
</table>

As you can see, there are significant differences between the 29 and 31 mi/hr tests. Clearly, it is important not to minimize the effect of speed when friction/drag factor tests are being done.

Test Skids With a Pavement Spotter and Radar

Often it is not easy to detect when a skid starts. The beginning of a skidmark is not nearly as dark as it becomes later in the skid. Topic 817 has a fairly long discussion about the difficulty of detecting skidmarks; if you have questions regarding their physical appearance, consult that topic. A pavement spotter can be used to determine the beginning of the deceleration of a vehicle doing a test skid. The spotter is sometimes referred to as a bumper gun (see Exhibit 10). A pigment is shot onto the pavement when the brakes are first pushed. This can be actuated by the brake light or some other electrical or inertial device. In this case, you may be adding deceleration distance to that caused by the full lock-up condition of the wheels. The effect of this is to calculate a lower drag factor, which will give a lower speed estimate of the accident vehicle.

Another way to increase your accuracy when doing test skids is to use radar to get the speed when the brakes are first applied. Most police agencies have ready access to such equipment. Both on-board and external radar units are very useful in making skid tests.

Test Skids With Accelerometers

Several accelerometers are available at a price that is affordable (less than $400) to many agencies that do test skids. Two such accelerometers are shown installed in cars in Exhibits 11 and 12. The model in Exhibit 11 is easy to install and gives peak and average g's in the longitudinal direction. It also gives the total time of deceleration and distance. Exhibit 12 shows a unit that can record up to eight minutes of data. It measures both longitudinal and lateral acceleration. Data from the accelerometer memory unit can then be transferred to a microcomputer.

Accelerometers are particularly helpful when a test must be made on a road with considerable traffic. Excepting the police, most people cannot close the road to traffic to do test skids. The accelerometer allows you to wait until there is a break in traffic, accelerate the car to the appropriate speed, check again for traffic, and slam on the brakes. When the vehicle stops, immediately move it off the roadway; the accelerometer will hold the data in its memory. After getting off the road, look at your data. This is certainly safer than trying to do the test when traffic is using the road. It is not necessary, of course, to measure the length of skids when you use the accelerometer. The deceleration is recorded in the unit. If there is simply too much traffic to do the test, use common sense and do not attempt it.

Sliding Test Tire

There are different methods that can be used to devise a test by dragging a test tire. A whole tire and wheel can be used as shown in Exhibit 13. Another method is to use part of a tire and fill it with concrete as shown in Exhibit 14. Both methods are often criticized because the load on the tire is considerably less than what a normal passenger car tire will
Exhibit 12. This model of accelerometer allows several minutes of data to be stored. Later it can be transferred to a personal computer for printing or storage.

be carrying. It is generally accepted that the friction capability of a rubber tire on pavement tends to deteriorate as the load is increased. It is also argued that at typical skidding speeds, the tire is heated due to friction. However, it is not expected that the heating of the pulled tire (or part of a tire) will have the same effect as the heating of a tire on a car that is being skidded. Thus, it is generally expected that friction data collected in this manner should be reduced to reflect this. Indeed, one commercial drag sled manufacturer suggests reduction factors.

If you are in a location where skidding a car is inherently dangerous, using a test tire could be appropriate. Sliding a test tire might not be appropriate, however, on soft turf, loose gravel, or other soft surfaces. A car skidding on such surfaces will normally have material build up in front of the sliding tires; whereas the sliding test tire may not weigh enough to

Exhibit 13. Coefficient of friction can be measured by dragging a tire of known weight along the pavement and measuring the pull required. You must make corrections of the angle of pull.
cause such build-up, resulting in lower friction coefficient values. On the other hand, it has been reported that a test tire sliding on grass can give a higher reading than a car sliding on grass. The reason for this is not clear. Perhaps there is some interaction of the tire and grass at a light load that does not occur with typical wheel loads.

Highway Department Skid Numbers

State highway departments (departments of transportation) regularly test their roads for friction coefficients. The tests are to determine skid numbers. A skid number is obtained by using an ASTM (American Society for Testing and Materials) tire in a specially designed drag trailer. The skid number is simply the friction coefficient obtained with these standard tests, multiplied by 100. Often the highway department will have done a test in the area of an accident. The question that sometimes comes up is whether the skid number represents the friction coefficient that a normal passenger car could be expected to get. In the Ohio tests (Reference 1), ASTM skid numbers were compared to calculated friction coefficients. At lower speeds it was concluded that the calculated friction coefficients were well above the ASTM skid numbers. However, there was generally good agreement at the higher speeds. This was partially attributed to the ASTM skid number test speed being at 40 mi/hr.

Several arguments against using highway department ASTM skid numbers in actual accident cases have been made. One is that the standard test tire does not have the same characteristics as typical passenger cars. It is generally argued that the ASTM tire will give lower friction values. Another argument is that the highway department tests may be dated and the surface condition has somehow changed. Also, the load on the tire may not approximate the load a normal passenger car would have. Thus, it may be just as well to do your own tests with an exemplar vehicle to eliminate these criticisms. However, if you get the case three years after the accident date and the road has since been repaved, a highway department skid number obtained near the accident date can be a useful place to start.

Table of Coefficients of Friction

Exhibit 15 gives a range of friction coefficients for several surface descriptions. The values given in Exhibit 15 are for passenger cars and pickup trucks equipped with typical tires. They are not typical values seen for heavy trucks. (See Topic 878 for typical deceleration values for heavy trucks). If tests cannot be made at an accident site, it may be necessary to use a range of typical values that have been seen for your surface description. Pavements described in the same words may have a considerable range of friction characteristics, depending on the exact material used and the detailed characteristics of the surface. The figures given are average coefficients of friction throughout the skid.

It can be seen in Exhibit 15 that higher friction coefficients are suggested for the lower speed range. This is consistent, of course, with the findings shown in Exhibit 4. For conservative speed estimates, use the lower coefficients given in the table. Also, you can see there is considerable difference between wet and dry surfaces. In nearly all cases, friction coefficients decrease considerably with wet surfaces. One exception is loose gravel. It is not uncommon to get higher friction coefficients with wet, loose gravel. The single most important factor affecting tire friction force in practice is the presence of water (in its various forms). The effect of water, snow, and ice is clearly evident by inspecting Exhibit 15.

Generally, it is very rare to see a friction coefficient much over 0.90. Exhibit 15 does list friction coefficients over 0.90 and even as high as 1.2. The value of 1.2 or even 1.0 is rarely experienced. If you have this high friction coefficient when you run a skid test, expect to see a great deal of abrasion to the tires on your test vehicle — and make sure that you are getting the full length of the test vehicle deceleration. If you are not using a pavement spotter (bumper gun), put a chalk mark or a piece of white tape on the sidewalls of your tires and have someone watch the marked wheels to see when they lock. Compare this location to the skidmarks left on the road. If they are different, make the appropriate corrections.

7. VELOCITY EQUATIONS FOR SKIDDING

At the beginning of this topic, equation (1) was given to calculate initial braking velocity when the distance the vehicle decelerated (d), the deceleration of the vehicle (a, negative for slowing), and the end velocity (v_e) are known. Equation (1) is repeated here:

\[ v_i = \sqrt{v_e^2 - 2ad} \]  

(1)
COEFFICIENTS OF FRICTION OF VARIOUS ROADWAY SURFACES

<table>
<thead>
<tr>
<th>DESCRIPTION OF ROAD SURFACE</th>
<th>DRY</th>
<th>WET</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Less than 30 mph</td>
<td>More than 30 mph</td>
</tr>
<tr>
<td></td>
<td>From</td>
<td>To</td>
</tr>
<tr>
<td>PORTLAND CEMENT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New, Sharp</td>
<td>.80</td>
<td>1.20</td>
</tr>
<tr>
<td>Traveled</td>
<td>.60</td>
<td>.80</td>
</tr>
<tr>
<td>Traffic Polished</td>
<td>.55</td>
<td>.75</td>
</tr>
<tr>
<td>ASPHALT or TAR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New, Sharp</td>
<td>.80</td>
<td>1.20</td>
</tr>
<tr>
<td>Travelled</td>
<td>.60</td>
<td>.80</td>
</tr>
<tr>
<td>Traffic Polished</td>
<td>.55</td>
<td>.75</td>
</tr>
<tr>
<td>Excess Tar</td>
<td>.50</td>
<td>.60</td>
</tr>
<tr>
<td>GRAVEL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed, Oiled</td>
<td>.55</td>
<td>.85</td>
</tr>
<tr>
<td>Loose</td>
<td>.40</td>
<td>.70</td>
</tr>
<tr>
<td>CINDERS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed</td>
<td>.50</td>
<td>.70</td>
</tr>
<tr>
<td>ROCK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crushed</td>
<td>.55</td>
<td>.75</td>
</tr>
<tr>
<td>ICE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth</td>
<td>.10</td>
<td>.25</td>
</tr>
<tr>
<td>SNOW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packed</td>
<td>.30</td>
<td>.55</td>
</tr>
<tr>
<td>Loose</td>
<td>.10</td>
<td>.25</td>
</tr>
</tbody>
</table>

Exhibit 15. This table lists coefficients of friction of various roadway surfaces. This table is not intended for large, heavy trucks.

Of course, acceleration is related to drag factor by equation (2):

\[ a = fg \] (2)

Again, remember that if a vehicle is slowing, you must insert a negative sign, -, in front of the value for acceleration.

A simple example problem follows. Assume a car skidded to a stop \( (v_e = 0) \) with a drag factor \( (f) \) of 0.75 over a distance \( (d) \) of 95 ft. The acceleration, \( a \), is calculated from equation (2):

\[ a = fg \] (2)

\[ = - (.75)(32.2 \text{ ft/sec/sec}) \]
\[ = - 24.2 \text{ ft/sec/sec} \]

The initial velocity is calculated from equation (1):

\[ v_i = \sqrt{v_e^2 - 2ad} \] (1)

\[ = \sqrt{0^2 - 2(-24.2)(95)} \]
\[ = 67.8 \text{ ft/sec} \]

Validation of Equation (1)

At times the application of equation (1) to accident reconstruction has been questioned. One of the objectives of the study listed as Reference 1 was to look at equation (1) coupled with equation (2) to determine whether other equations could be more accurate. The NHTSA researchers concluded that the
use of the theoretical equations, (1) and (2), is valid; they were the most accurate of the equations tried. The researchers further concluded that the assumptions used in the theoretical equations are valid. Concerns such as constant deceleration throughout the skid were shown to be essentially correct. The difference in deceleration throughout the skid was found to be too small to affect the results. They did point out the concern to do your test skids at a similar speed. Once wheels lock, they stay locked, leaving marks. Not always would a wheel lock. This was found not to cause problems during hard (panic) braking. The effect of other forces, such as aerodynamic drag, was found to be insignificant when compared to the braking force. Finally, the slide friction coefficient of a wheel was found to depend only on the vehicle's overall weight, tire type, and pavement composition (as long as the speed effect was taken care of as stated before). Variations in at-rest loads on the tires, weight shift, and temperature were found to be insignificant.

### Tables and Nomographs to Determine Skidding Vehicle Speeds

For those not used to using equations, tables have been developed to determine the initial speed of vehicles that skid to a stop. Typical tables are shown as Exhibits 16 and 17. Exhibit 16 has been worked in the units of mi/hr. To use the table you must first determine the drag factor and length of the slide to stop in feet. You need to know the same variables to use Exhibit 17. However, Exhibit 17 requires that the length of the slide to stop be measured in meters, with the resulting speed in km/hr.

Another method that can be used to determine the initial speed of a vehicle that skids to a stop is to use a nomograph. Two such nomographs are Exhibits 18 (for U.S.A. units) and 19 (for metric units). Put a straight edge on the appropriate drag factor and distance at which the vehicle slid to a stop. Where the straight edge intersects the speed line, read the initial speed in either mi/hr or km/hr.

### Unequal Drag Factors on Axles

Passenger cars do not have the same load on each axle when they are not being accelerated (i.e., when at rest or moving at constant velocity). During hard braking, load is shifted to the front axle. Because of this load shift, no correction needs to be made when all wheels are locked. If, however, one axle has a drag factor different from that of the other axle, you cannot assume the car was braked with a drag factor equal to locked wheel conditions (that is, drag factor for the whole vehicle equal to the sliding coefficient of friction). Equation (6) addresses this (see Topic 890 for the derivation):

\[
    f_R = f_t - x_t (f_t - f_r) \frac{1 - z(f_t - f_r)}{1 - z(f_t - f_r)}
\]

where \( f_R \) = drag factor on the vehicle, \( f_t \) = drag factor on the front axle, \( f_r \) = drag factor on the rear axle, \( x_t \) = horizontal distance of the center of mass from the front axle as a decimal fraction of the wheelbase, and \( z \) = height of the center of mass as a decimal fraction of the wheelbase.

Consider the following example. Assume the height of the vehicle's center of mass is 2.08 ft and is located 4.22 ft behind the front axle. The wheelbase of the car is 9.39 ft. Therefore, \( z \) is equal to \( 0.222 \times (2.08/9.39) \) and \( x_t \) is equal to \( 0.449 \times (2.08/9.39) \)

Therefore, the drag factor for the whole vehicle is 0.26.

### Drag Factors for Other Vehicles

This topic does not discuss how to determine drag factors for motorcycles and articulated vehicles. These vehicle types are discussed in Topics 874 and 878. Unlike passenger cars and pickup trucks, the operators of motorcycles and articulated vehicles generally have the option of applying brakes on only some of the axles. Therefore, do not assume that the drag factors for motorcycles and large trucks are the same as the values that can be obtained for four-tired vehicles.

### Drag Factor on a Grade

For the case where a vehicle is skidding with all wheels locked on a non-level surface (that is, up or down a grade), the drag factor is not equal to the coefficient of friction. In Topic 890 it was shown that the drag factor on a grade is given by the following formula:

\[
    f_G = (\mu + G) \sqrt{1 + G^2}
\]
## Exhibit 16

**U.S.A. Units**

### SPEED IN MILES PER HOUR REQUIRED TO SLOW TO A STOP

For various distances and surfaces:

<table>
<thead>
<tr>
<th>Length of Slide</th>
<th>Ice</th>
<th>Snow</th>
<th>Gravel</th>
<th>Clean, wet paving</th>
<th>Clean, dry paving</th>
<th>Fair brakes</th>
<th>Good brakes</th>
<th>Excellent brakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The table shows the speed in miles per hour required to slow to a stop for various distances and surfaces, with different braking conditions.
## Exhibit 17

**Metric Units**

### SPEED IN KILOMETERS PER HOUR REQUIRED TO SLOW TO A STOP

**FOR VARIOUS DISTANCES AND SURFACES**

<table>
<thead>
<tr>
<th>Length of skid to stop (meters)</th>
<th>Ice</th>
<th>Snow</th>
<th>Gravel</th>
<th>Clean, wet paving</th>
<th>Clean, dry paving</th>
<th>Fair brakes</th>
<th>Good brakes</th>
<th>Excellent brakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>0.10</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>0.20</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>0.30</td>
<td>9</td>
<td>12</td>
<td>14</td>
<td>15</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>0.40</td>
<td>11</td>
<td>16</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>22</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>0.50</td>
<td>13</td>
<td>18</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>0.60</td>
<td>15</td>
<td>20</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>0.70</td>
<td>17</td>
<td>22</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>0.80</td>
<td>19</td>
<td>24</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>0.90</td>
<td>21</td>
<td>25</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>0.95</td>
<td>23</td>
<td>27</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>1.00</td>
<td>25</td>
<td>28</td>
<td>30</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>1.10</td>
<td>27</td>
<td>29</td>
<td>31</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>1.20</td>
<td>29</td>
<td>30</td>
<td>32</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

---

**Note:** The table provides the recommended speed in kilometers per hour required to slow to a stop for various lengths of skid marks. The table includes columns for different types of surfaces and braking conditions, as well as for different types of brakes.

---

**Source:** [Source for the table](#)

---

**Figure:**

- **Legend:**
  - Ice
  - Snow
  - Gravel
  - Clean, wet paving
  - Clean, dry paving
  - Fair brakes
  - Good brakes
  - Excellent brakes

---

**Legend Notes:**

- **Ice:** The table provides the speed required for ice conditions.
- **Snow:** The table provides the speed required for snow conditions.
- **Gravel:** The table provides the speed required for gravel conditions.
- **Clean, wet paving:** The table provides the speed required for clean, wet paving conditions.
- **Clean, dry paving:** The table provides the speed required for clean, dry paving conditions.
- **Fair brakes:** The table provides the speed required for fair brakes conditions.
- **Good brakes:** The table provides the speed required for good brakes conditions.
- **Excellent brakes:** The table provides the speed required for excellent brakes conditions.

---

**Additional Information:**

- **Exhibit Notes:**
  - The exhibit provides a comparison of the required speeds for different braking conditions and surface types.
  - The exhibit is designed to aid in traffic management and safety planning.

---

**Conclusion:**

The exhibit provides a comprehensive overview of the required speeds for various braking conditions and surface types, which can be used to improve traffic management and safety planning.
Exhibit 18.

U.S.A. Units

NOMOGRAPH RELATING STOPPING DISTANCE AND TIME, INITIAL SPEED, AND AVERAGE DRAG FACTOR

U.S.A.
where \( f_G \) = drag factor on a grade, 
\( \mu \) = coefficient of friction between the tires and pavement, and 
\( G \) = the percent grade expressed as a decimal (positive (+) for an upgrade and negative (−) for a downgrade).

Consider the following example. Assume the coefficient of friction, \( \mu \), is equal to 0.80 and the vehicle in question is sliding down a 4 percent grade \((G = -0.04)\). Substituting into equation \((7)\), the effective drag factor on a grade is calculated as follows:

\[
f_G = \frac{\mu + G}{\sqrt{1 + G^2}} \quad (7)
\]

\[
= \frac{0.80 - 0.04}{\sqrt{1 + (-0.04)^2}}
\]

\[
= 0.76/1.0016
\]

\[
= 0.76
\]

As you can see in this example, the denominator is so close to 1.0 that there is little reason to include it in the calculation. Because of this, equation \((7)\) is normally shortened to the following:

\[
f_G = \mu + G \quad (8)
\]

For the example that was just worked, there is no significant difference whether equation \((7)\) or \((8)\) is used. With a downgrade or upgrade as high as 12 percent \((G = -0.12 \text{ or } 0.12)\), there is no significant difference between the answers obtained by using either equation. A grade of 12 percent is hardly ever seen on a typical roadway. Thus, for nearly all cases, the use of equation \((8)\) is very acceptable.

8. SPEED ESTIMATES FROM IRREGULAR SKIDMARKS

Four different patterns of skidmarks are encountered in examining the roadway after an accident:

1. No sign of skidmarks is found, although there may be other kinds of tire marks. In this case, there is no indication on the road of any braking whatsoever. This does not mean there was no braking, only that braking was insufficient to lock wheels or that skidmarks disappeared before they were observed. There may be other indications of braking, such as the reports of witnesses. The car might have been equipped with an anti-lock braking system (ABS). With ABS-equipped cars, skidmarks may or may not be seen.

2. Skidmarks of nearly the same length are made by all wheels. No mark is more than 5 percent longer than the shortest mark. Such marks indicate that all wheels were locked suddenly and that the whole vehicle slid. This is a common circumstance. Unless there is reason to believe otherwise, one may conclude that the wheels were locked by braking. Use the longest skidmark and a drag factor based on coefficient of friction to estimate speed as it has already been described.

3. There are definite skidmarks from all wheels, but they vary considerably in length.

4. Some, but not all, wheels make skidmarks.

The last two of these cases are irregular skidmarks.

To estimate speeds from skidmarks, you need to know values for drag factor (deceleration) and the distance through which it operated. In Case 1, there were no skidmarks, so no estimate of speed from skidmarks is possible. In Case 2, the length of the longest skidmark indicates the distance and the fact of sliding determines the drag factor, so estimating the speed the vehicle had to be going when it started to slide to a stop is a simple matter. For Cases 3 and 4 (irregular skidmarks), adjustments have to be made to get the equivalent complete skid or an equivalent reduced drag factor.

*Two different assumptions* are commonly made in speed estimates based on irregular skidmarks:

1. Average skidmark length is assumed to be the equivalent all-wheel sliding distance.
2. The longest skidmark is assumed to be the equivalent all-wheel sliding distance.

In both cases, pavement coefficient of friction is the basis for drag factor estimates and calculations. *Implications* of whatever assumptions are made must be understood to be able to appraise the significance of the estimate based on them. These implications will be discussed and illustrated by applying each assumption to the same example of a set of irregular skidmarks.

**Skidmarks from All Wheels**

If all wheels do eventually leave genuine skidmarks, two conclusions are immediately warranted:

1. Brakes were capable of locking all wheels.
2. The driver was able to apply sufficient pedal pressure to cause them to do so.

As brake pedal pressure increases for one or more of several reasons, some wheels lock before the others. *Differences in pavement* are sometimes responsible. Other things being equal, the wheel with the least pavement coefficient of friction will lock first as brake pedal pressure increases. A pavement may have greater coefficient of friction in some places than in others because of wear; because of foreign material such as loose dirt, oil spatter, or moisture; or because of different surface material, for example in patched paving.

*Differences in weight on wheels* may also account for some wheels skidding and others not. Other things being equal, the wheel with the least load on it will lock first as brake pedal pressure is increased. Differences in static load are produced in two ways:

1. Uneven loading of passengers or freight;
2. Crossways tilt of the road, which shifts weight from the uphill to the downhill side.
Unequal load, front and rear, due to weight shift (dynamic load) in braking may be compensated for in vehicle brake design. As brake pedal pressure is increased, the wheel with least weight in proportion to its braking strength will lock first. Cars equipped with ABS brakes compensate automatically for differences in wheel loads.

**Differences in tire radius** may affect which wheel locks first. Other things being equal, the wheel with the least radius will lock first as brake pedal pressure is increased. The maximum torque (rotational moment) of a wheel on a vehicle in motion is the coefficient of friction of the tire on the road times the weight on the tire times the leverage (moment arm). The moment arm is the distance from the axle to the road — that is, the tire radius. With equal road friction, weight, and brake resistance to rotation, the wheel with the greatest moment arm can overcome brake resistance, while the wheel with the least moment arm cannot. Thus, with increasing but equal brake effort, the wheel with the least radius (moment arm) will be the first to start sliding. The rolling radius of a tire may be decreased by two things:

1. Tread wear;
2. Overdeflection due to overloading or underinflation or both.

If brakes are applied very quickly and very hard, brake force will almost instantly lock all wheels; but if braking is applied gradually, the wheel with the least resistance to locking locks first, followed in turn by the others. Such gradual brake application often produces skidmarks varying considerably in length.

In ordinary vehicle operation, brakes are usually applied gradually and rarely to the full extent of the capabilities of brakes and driver. In emergency braking, brakes may also be applied gradually because the driver does not, at first, perceive the need for maximum braking or because the driver is reluctant to do as much braking as he can.

**During partial skidding,** tires which are sliding are producing all the drag of which the pavement at that point is capable. The other tires, which have not yet started to slide, are also being braked and so are also producing a slowing drag on the vehicle. Indeed, a wheel which leaves no skidmark may produce more road drag but with less drag factor than one which is skidding. This may be due to greater weight on it or because the pavement under it has greater friction or both. These conditions give the tire a better grip on the roadway so it can use more of the brake system capability. Furthermore, the friction drag of the tire on the road is greatest just before the tire begins to slide (static friction compared to dynamic friction).

**Assumption 1 — Average Skidmark Length**

Using average skidmark length as the basis for estimating a slide-to-stop speed assumes that there is the equivalent of complete locked-wheel skidding for a distance equal to the average skidmark length, but no other braking. However, if all wheels eventually lock, all will be doing some braking — often much braking — from the instant brakes are applied. Thus, average skidmark length gives a low speed estimate.

Such a "conservative" speed estimate favors the defendant who is being tried for speeding. In civil cases, however, what benefits the defendant is disadvantageous to the plaintiff, and the "conservative" skidmark assumption may do as much harm as good.

**Example.** Exhibit 20 shows a set of four skidmarks varying considerably in length. To begin with, the investigator must be sure that these are really skidmarks, especially toward the end where the vehicle swerves. That they are indeed skidmarks is indicated by the fact that striations throughout the marks are parallel to the marks. (See Topic 817 for a detailed description of skidmarks).

The speed limit at this point was 55 mi/hr. The pavement was dry, clean, well-traveled Portland cement concrete and had a three percent downgrade \( G = -0.03 \) in the direction of travel.

The vehicle turned almost end for end during the last part of its approach to the collision point. This is because the rear wheels locked first. Having used all available traction by skidding, the rear wheels offered no more resistance to side-slipping downhill to the right on the cross slope of the pavement surface. The front wheels, not locked, had surplus friction which kept them from sliding downslope toward the right, resulting in the vehicle's assuming an angle to the roadway. The angle continued to increase rapidly until the vehicle was moving almost backward.

The vehicle was nearly stopped when it reached the tracks, where it was snagged by the passing train and carried off the roadway.

The four skidmarks measured:

<table>
<thead>
<tr>
<th>Location</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right rear</td>
<td>407 ft</td>
</tr>
<tr>
<td>Left rear</td>
<td>201 ft</td>
</tr>
<tr>
<td>Left front</td>
<td>115 ft</td>
</tr>
<tr>
<td>Right front</td>
<td>96 ft</td>
</tr>
<tr>
<td>Total</td>
<td>819 ft</td>
</tr>
</tbody>
</table>

Average 205 ft

Lacking test skids, the coefficient of friction range given in Exhibit 15 can be used. For high speed on the pavement described, this would be 0.60 to 0.75. With a grade of –0.03, the drag factor would be between 0.57 and 0.72 for all wheels locked.

For an equivalent sliding distance of 205 ft and a drag factor of 0.57 to 0.72, Exhibit 16 gives slide-to-stop speeds of 59 to 66 mi/hr. Because this method gives an inherently low estimate, the best conclusion might be that the speed at the beginning of the slide was definitely more than 59 mi/hr.

Some people who use average skidmark length in estimating speed justify this method by pointing out that the average
of several measurements is more reliable than any of the individual measurements. This is a spurious explanation, because measuring the lengths of four skidmarks is not the same as four measurements of the same skidmark.

**Assumption 2 — Longest Skidmark**

Using the longest skidmark as a basis for estimating speed assumes that there was braking on all wheels throughout the length of the longest skidmark, and that braking on wheels that did not make marks was as much as it would have been if the tire had been sliding. The fact that wheels eventually locked indicates that all wheels had braking. As explained earlier, it is possible that a wheel that left no skidmark was actually developing more pavement drag — and so contributing more to slowing the vehicle — than another wheel on the same vehicle which was sliding and making a mark.

**Example.** Apply the longest-skidmark assumption to the skidmarks illustrated in Exhibit 20. Using the appropriate equations \( v_f = \sqrt{v_i^2 - 2ad} \) and \( a = \frac{g}{f} \), the longest mark, 407 ft, gives a brake-to-stop speed of 83 to 94 mi/hr. Because the longest-skidmark method gives an inherently high estimate, the best conclusion might be that the speed at the beginning of the slide was definitely less than 94 mi/hr.

**Comparison** of the two assumptions is as follows:

1. Average skidmark — more than 59 mi/hr
2. Longest skidmark — less than 94 mi/hr

Using both assumptions, you would conclude that the brake-to-stop speed was between 59 and 94 mi/hr.

Part of this combined estimate range is due to using a range of values for coefficient of friction. If careful examination of the road surface or skillfully made test skids provide a lesser range of values for coefficient of friction, the estimate could be somewhat more precise.

Of course, if the vehicle did not slide to a stop but was stopped by a collision, minimum and maximum estimated speeds would both be greater.

Such a wide range of estimated values for brake-to-stop speed could make the estimate almost useless. But if the low value obtained from average skidmark length or the high value obtained from the longest skidmark serve the purpose for which the estimate was made, the estimate is satisfactory.

**More Precise Estimate**

If the simple assumptions just described are inadequate, more precise estimates with a lower range of values can be made. A more accurate estimate can be illustrated by applying the procedure to the same example, Exhibit 20. For this purpose, the same range of values for coefficient of friction will be continued, with the understanding that the estimate range might be further narrowed by achieving a lesser range of values for coefficient of friction.

**Center of mass movement.** If the vehicle rotates while skidding, as in Exhibit 20, the longest skidmark exaggerates the distance the vehicle as a whole slid while rotating. Then it is better to consider the length of skid as the distance moved by the vehicle's center of mass. In the example, the longest skidmark is 407 ft, but the center of mass movement measures 395 ft — that is, the vehicle moved 395 ft while skidding. Using this distance, the brake-to-stop speed estimate becomes 82 to 92 mi/hr. The reduction from the 83-to-94 mi/hr range is not much for such a long skid, but it is more accurate. The difference would be greater for a shorter total skidding distance.

Dividing the skidding distance into sections and applying equation (1) below (working backward from the vehicle's final position) will give a better final estimate than either of the assumptions heretofore described.
More careful analysis of the rate of slowing as determined by the coefficient of friction during movement in each section will also contribute to the reliability of the estimate. Refer again to the example, Exhibit 20. There are four sections, as follows:

1. A to B 202 ft — one skidmark
2. B to C 67 ft — two skidmarks
3. C to D 22 ft — three skidmarks
4. D to E 104 ft — complete skidmarks

These are center-of-mass distances. Their sum is 395 ft.

Begin with the simplest section, D to E. During these 104 ft, the vehicle was completely sliding with all four wheels. Calculating the slide-to-stop speed for this distance, therefore, is not difficult. The usual equation, with the previously used drag factor range of 0.57 to 0.72, gives a slide-to-stop speed estimate of 42 to 47 mi/hr.

The remaining incomplete skids are more complicated. Consider the movement from B to C, 67 ft. It is clear that both rear wheels are sliding. Therefore, the rear axle develops the full drag factor provided by roadway friction. But the rolling front wheels are not developing that much drag factor, despite their being on the same surface. Front wheel drag factor is therefore less than pavement coefficient of friction, but certainly much more than rolling friction, because beyond Point C the front wheels begin to skid. Now a drag factor or range of drag factors between rolling plus air resistance and maximum roadway drag factor must be chosen. A reasonable assumption would be that the non-sliding front wheels developed at least half of the minimum roadway drag factor — that is, 0.57/2 = 0.29 — and as much as nine tenths of the maximum roadway drag factor — that is, (0.9)(0.72) = 0.65. The half and nine tenths suggested here are simply matters of judgment on the part of the investigator. On the basis of what you know about the situation, you must choose values or limits which can reasonably be expected to embrace the range of possibilities. This gives the following reasonable values for front and rear axle drag factors:

<table>
<thead>
<tr>
<th>Section B to C</th>
<th>Axle</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front, $f_f$</td>
<td>0.29</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Rear, $f_r$</td>
<td>0.57</td>
<td>0.72</td>
<td></td>
</tr>
</tbody>
</table>

These values can now be applied as described for irregular braking. Assume, for example, that the vehicle involved has a wheelbase $l = 10$ ft, that its center of mass is $x_l = 4.3$ ft behind the front axle, and that the center of mass is $z = 2.0$ ft above the road surface. Then $x_r = 0.43$ and $z = 0.20$.

The equation for resultant total drag factor when axles have different drag factors is

$$ f_R = f_f - x_f (f_f - f_r) $$

Substitute the vehicle constants in this equation for repeated use:

$$ f_R = f_f - 0.43(f_f - f_r) $$

Substitute the minimum axle drag factors for this particular section of the skid, noting that $f_r - f_f = 0.29 - 0.57 = -0.28$:

$$ f_R = \frac{0.29 - 0.43(-0.28)}{1 - 0.20(-0.28)} $$

$$ f_R = 0.404/1.056 $$

$$ f_R = 0.39 $$
Thus, the minimum drag factor with the assumptions noted is equal to 0.39. Repeat this for the maximum axle drag factor. The result is $f = 0.67$, maximum.

Movement from section A to B is made with only the right rear tire sliding. That wheel would have the full roadway drag factor. Use the same values for reduced drag factor as before. The front axle would then have the same drag factor range as for section B to C, namely, 0.29 to 0.65. The left rear wheel, not sliding, would have the same range, but the right rear wheel, sliding, would have the drag factor range of the pavement. The rear axle drag factor would be the average of the rear wheel drag factors — that is, a minimum of $(0.29 + 0.57)/2 = 0.43$ and a maximum of $(0.65 + 0.72)/2 = 0.69$. This gives the following reasonable values for front and rear axle drag factors. The calculated corresponding values for the whole vehicle are also shown.

<table>
<thead>
<tr>
<th>Section A to B</th>
<th>Axle</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front, $f_f$</td>
<td>0.29</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Rear, $f_r$</td>
<td>0.43</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Vehicle</td>
<td>0.34</td>
<td>0.66</td>
<td></td>
</tr>
</tbody>
</table>

The remaining section, C to D, can be calculated in the same way:

<table>
<thead>
<tr>
<th>Section C to D</th>
<th>Axle</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front, $f_f$</td>
<td>0.43</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Rear, $f_r$</td>
<td>0.57</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Vehicle</td>
<td>0.48</td>
<td>0.70</td>
<td></td>
</tr>
</tbody>
</table>

The initial velocity of the vehicle can now be calculated, because the drag factor (or acceleration value) is now known for each section. Use the equation

$$v_i = \sqrt{v_e^2 - 2ad} \quad (1)$$

to calculate the end velocity of the preceding section. Remember that $a = fg$. For these calculations, $a$ will always be negative. For example, use the distance and drag factors to calculate the initial speed at D, using the distance and acceleration values from D to E. The initial velocity calculated at D is the end velocity for the C to D section. Then use that end velocity for the C to D section to calculate the initial velocity at C. Continue this procedure until the initial velocity is finally calculated at A. This procedure is summarized in Exhibit 21.

<table>
<thead>
<tr>
<th>Section</th>
<th>Distance (ft)</th>
<th>Drag factor (Min)</th>
<th>Initial velocity (ft/sec)</th>
<th>End velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D to E</td>
<td>104</td>
<td>0.57 to 0.72</td>
<td>61.8 to 69.4</td>
<td>0.0 to 0.0</td>
</tr>
<tr>
<td>C to D</td>
<td>22</td>
<td>0.48 to 0.70</td>
<td>67.1 to 76.2</td>
<td>61.8 to 69.4</td>
</tr>
<tr>
<td>B to C</td>
<td>67</td>
<td>0.39 to 0.67</td>
<td>78.6 to 93.3</td>
<td>67.1 to 76.2</td>
</tr>
<tr>
<td>A to B</td>
<td>202</td>
<td>0.34 to 0.66</td>
<td>102.9 to 131.5</td>
<td>78.6 to 93.3</td>
</tr>
<tr>
<td>Overall</td>
<td>395</td>
<td>0.41 to 0.68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 21. Summary of example of velocity estimate of incomplete skidmarks illustrated in Exhibit 20.

The table shows the initial velocity at A to be between 102.9 and 131.5. Round off these velocities to between 103 ft/sec and 132 ft/sec. The average drag factor for the entire 395 ft range between 0.41 and 0.68. The velocity in ft/sec changed to the units of mi/hr gives a range of 70 to 89 mi/hr.

Although an estimate of 70 to 89 mi/hr may seem like a large range, it is certainly much better than the minimum of 59 mi/hr obtained by averaging skidmarks and the maximum of 94 mi/hr from the longest single tire mark. From the more precise estimate of a minimum of 70 mi/hr, you can conclude with confidence that the vehicle was exceeding the speed limit of 55 mi/hr.

Further adjustment might be made in the precise estimate. The method outlined assumes that the drag factor is constant throughout each section of the slide. This means that at the end of each section of skidding, the drag factor would have to increase instantly to a greater value as another wheel begins to skid. This is, of course, impossible, especially because the pattern of the skidmarks strongly suggests that the driver steadily increased brake pressure during the approximately 8 sec of skidding. It would be more accurate to assume that the estimated drag factor for, say, section A to B was at A and that the drag factor increased steadily to B. The effective drag factor for the A to B section would be the average of the drag factors at A and B. For example, Exhibit 21 shows that the minimum drag factor assigned to the A to B section is 0.34, and that for the B to C section is 0.39. If these are the values at the beginning of each section, the more precise drag factor for A to B would be $(0.34 + 0.39)/2 = 0.365$.

Furthermore, if brakes are applied gradually, there would undoubtedly have been brake application before any tires started to slide. This braking would have had an average drag factor between zero and the beginning of skidding at A, where it is assigned a value of 0.34. That makes an initial non-skid distance with an average minimum drag factor of $(0 + 0.34)/2 = 0.17$. The distance is indeterminate.

Both of these adjustments increase drag factor estimates in all but the last section and so increase the total brake-to-stop speed estimate. This increase would be small, and in view of the broad range of possible values assigned to pave-
ment coefficients of friction, would add little to the speed estimates. Omitting such an adjustment makes the speed estimate, both minimum and maximum, slightly on the low side. Another refinement might also be considered. In estimating axle drag factors for the example, no distinction was made between right and left wheels. That is the same as assuming that right and left wheels were equally loaded. The fact of cross slope in the vehicle's lane because of road crown means that, even if the vehicle has equal weight on right and left sides, the right wheel will carry a little more load than the left. This can produce two effects on skidmarks which can only be distinguished by close observation:

1. Both right and left tires on the axle slide, but the one with less load heats so little that the mark it makes is faint and may go unnoticed.
2. The wheel with more load, usually on the downhill side, grips the roadway a little better and so does not lock, whereas the opposite wheel on the axle, with less load, does slide and leave a mark.

In the example discussed, Exhibit 20, the uphill left tire at first makes no mark that was reported. Perhaps that tire was also sliding but making a mark too light to be observed, or perhaps it carried enough weight because of lopsided loading to grip the pavement better than the right downhill tire. It is also possible that the road surface had different characteristics for right and left tires, that the brakes were somewhat different in their rate of application, or that right and left tires had a somewhat different rolling radius. These are imponderables which must be considered even if they cannot be evaluated. In the example, the cross slope of the traffic lane is small and the conclusion is warranted that the effect of these circumstances would be small and surely within the broad range of the speed estimate produced by the range of values chosen for pavement coefficient of friction.

Judgment is required in choosing limits of values to make speed estimates, as it is in many other kinds of engineering. It is important that the ultimate estimate have sufficient range reasonably to embrace conceivable variables. In the illustrated example and in most similar problems, the judgments involve consideration of such matters as:

- Selection of a range of roadway coefficients of friction
- Selection of values of drag factor for wheels that are braking but make no tire marks
- Consideration of possible rates of brake application
- Evaluation of reliability of reported observations and measurements of tire marks on the road
- Interpretation of possible effect of distribution of load and cross slope on roadway.

A lesser range of speeds in the final estimate is most easily achieved by more careful evaluation of coefficient of friction. Well made test skids or skillful examination of the road surface may yield a lesser range of values than those suggested by Exhibit 15.

It is also possible that skidmarks from some wheels may be so faint or obscure as to escape notice and so go unreported. Occasionally those matters can be cleared up by careful examination of photos.

Some Wheels Making No Skidmarks

If some but not all wheels make skidmarks, other considerations are involved. Two circumstances are possible:

1. All wheels were braking, but some not sufficiently to lock the wheel.
2. Some wheels were doing no braking at all.

If some wheels make no skidmarks when others do, it is important to discover whether these wheels were without brakes. There are three possibilities:

1. A wheel has no brake — for example, a wheel on a small trailer or a wheel on the front axle of a tractor and semi-trailer.
2. A wheel has a brake, but it was not used — for example, the front wheel on a motorcycle or a wheel on a tractor-trailer combination with independently controlled trailer brakes.
3. A brake fails to function.

Which wheels have no brakes can be ascertained ordinarily by vehicle inspection or by statements from the vehicle operator or owner. Whether independently operated brakes were used can sometimes be discovered by questioning the driver. Sometimes careful mechanical examination will reveal that a brake could not have functioned. Driver statements are not to be trusted with respect to whether brakes failed to function.

If it is clear that a wheel was without braking, its drag factor is only that of rolling resistance, which may be taken as 0.01 or simply omitted altogether. Therefore, it is possible to ascribe a very small but definite drag factor to wheels without brakes. Then the matter can be handled as described for irregular braking, in which wheels and axles have different drag factors and weight shift can be taken into consideration. This is essentially what was done for the skid from A to D in Exhibit 20 where some wheels made no skidmarks.

If wheels with optional braking make skidmarks, the skidmarks are proof that brakes were applied; the skidmarks establish a drag factor for the wheel based on the roadway coefficient of friction. If the skidmarks made by such wheels are approximately as long as skidmarks from other wheels, treat the case as one of complete braking with all wheels locked and tires sliding. If the mark made by a wheel with optional braking is shorter than the others, the brake on that wheel was apparently applied later or sooner than the others. Consider as a complete skid the distance the vehicle moved while all wheels made skidmarks. The remaining distance is complicated, because there is no way to know how much the optional brake was applied. The drag factor for the nonskidding wheel
must then be handled as a range of values, possibly as little as rolling resistance drag factor and as much as full skidding. The exact values may be governed by what has been learned from any source about the possible application of brakes. Thus, the partial braking may be treated as two sections: one distance in which skidding is complete, with a drag factor based on the coefficient of friction; and the other distance an estimated possible range of drag factors. The calculated brake-to-stop speeds for the two sections are then combined in the usual manner. This usually gives a wide range of estimated speed, but there is no help for that.

Finally, if all brakes are applied by the same control and there is nothing to suggest brake malfunction, you can infer, if skidmarks are irregular, that all brakes were contributing to slowing. Then the matter can be handled like the first three sections of the skid described in Exhibit 20, perhaps with a greater range of drag factor for wheels which do not make skidmarks.

**Clues from skidmarks** may indicate to some extent what braking took place and so aid in assigning upper and lower limits for drag factors. If a skidmark is made from only one side of a vehicle and this mark is quite straight for 50 ft (15 m) or more, you can infer that there was about the same braking, more or less, on both sides of the vehicle. With significantly unequal braking, the vehicle would be expected to swerve. Without such a swerve, the drag factor of the wheels on the side where there was no mark can be considered about the same as the drag factor on the side which did mark. The possibilities of difference are reflected in the range of drag factors selected. Conversely, if there is a swerve with skidmarks from only one side of the vehicle, you can infer that braking was unequal. Then a range of possible drag factors can be assigned to reflect this difference with regard to the direction of the swerve. In this connection, you must be very careful to distinguish between braking skidmarks and steering yawmarks. (See Topic 817 for a discussion of the difference between the two).

Sometimes there are definite skidmarks from the right and left sides of the vehicle but it is not clear whether these are from front wheels, rear wheels, or both. If rear wheels track closely in the path of front wheels, it may not be clear where the marks from front or rear wheels begin. Careful examination of the marks or even detailed examination of photographs can settle questions of this kind.

If there is no way to find out how far individual wheels slid, you must assume a minimum and a maximum possibility which will give a greater range of speed in the final estimate. The most common circumstance of this kind is a report that information alone, you have no way of knowing whether

1. The measurements were made from the beginning of the rear tire marks to the end of the front tire marks, in which case the actual sliding was 50 feet minus the wheelbase of the vehicle.

2. The measurements were made from the beginning to the end of a rear or front tire mark, in which case 50 feet represents the length of a complete skid.

Both distances must be used as a basis for estimating, and this may add greatly to the range of the final brake-to-stop speed estimate.

If tire marks continue straight for 50 ft (15 m) or more, the front wheels can be assumed to have locked first because the rear wheels had locked while the front ones were rolling, the vehicle would probably have turned around during the last part of the skid as illustrated in Exhibit 20.

**Braking Distance**

If braking distance from a specified speed is wanted, estimates are less complicated because brake capabilities are stated. If sliding is involved, maximum and minimum drag factors need to be assigned only to pavement coefficient of friction. One of several levels of brake performance may be specified, such as:

- Brakes meet but do not exceed legal requirements. Drag factor is then determined by the law and by the roadway grade.
- Brakes lock all wheels. Drag factor is then determined by the roadway coefficient of friction and grade.
- Brakes lock certain specified wheels but not others. Resistant drag factor is then calculated as described for irregular braking. This requires special data on the vehicle as loaded.

Once the applicable drag factor has been determined, the brake-to-stop distance for the specified speed is found from the appropriate equation, table or nomograph.

**Skidmarks Leaving One Surface and Entering Another**

To determine the speed required to skid to a stop when skidmarks show that the vehicle slid from one surface to another, you need to know the coefficients of friction of the surface and, to be precise, the location of the vehicle's center of mass. Calculations are accomplished by dividing the total skidding distance into a number of sections, depending on how many wheels are on each surface. Start at the vehicle's rest position and work backward to its initial braking point using equation (1):

$$v_i = \sqrt{v_e^2 - 2ad} \quad (1)$$

You will recall that $a = fg$ and $a$ is negative for a skidding condition.

**An example.** Exhibit 22, illustrates how such an estimate is made. All wheels make skidmarks, and rear-wheel skidmarks follow front-wheel skidmarks closely. The pavement's coefficient of friction is taken as 0.75 to 0.90, that of the shoulder as 0.45 to 0.65. There is no grade.
The center-of-mass movement is divided into five sections as follows:

1. A to B 21 ft — entirely on roadway
2. B to C 10 ft — one wheel on shoulder
3. C to D 81 ft — two wheels on shoulder
4. D to E 10 ft — three wheels on shoulder
5. E to F 64 ft — entirely on shoulder

Section A to B, with all wheels sliding on the roadway, is calculated as a complete skid for 21 ft center-of-mass movement at \( f = 0.75 \) to 0.90, a somewhat narrower range than indicated in Exhibit 15.

In section E to F the vehicle slides entirely on loose gravel of the shoulder. The section drag factor is assumed to range between \( 0.45 \) to \( 0.65 \).

In section C to D both front and rear axles have one wheel sliding on the roadway and one on the shoulder; therefore, front and rear braking drag factors are the same. The effective drag factor for both axles and the whole vehicle is then the average drag factor for the two sides, which is \( f = (0.75 + 0.45)/2 = 0.60 \) minimum and \( f = (0.65 + 0.90)/2 = 0.78 \) maximum.

In section B to C both rear wheels are on the roadway, but one front wheel is on the roadway and the other is on the shoulder. Therefore, the front-axle drag factor is the coefficient of friction of the roadway, \( f_f = 0.75 \) to 0.90; but the front-axle drag factor is the average of the roadway and shoulder as calculated for section C to D, \( f_f = 0.60 \) to 0.78. Different front and rear drag factors call for computation of the resultant drag factor. For minimum braking, \( f_f = 0.60 \) and \( f_r = 0.75 \). Use the same vehicle data as in the example for skidmarks of different lengths, \( x_f = 0.43 \) and \( z = 0.20 \):

\[
 f_r = \frac{f_f - x_f (f_f - f_i)}{1 - z (f_f - f_i)}
\]

Calculate the maximum drag factor the same way. With \( f_f = 0.78 \) and \( f_r = 0.90 \), the resultant maximum drag factor is \( f = 0.81 \).

Section D to E is similar to section B to C. Both front wheels are on the shoulder, left rear wheel is on the roadway, and right rear wheel is on the shoulder. Therefore, the front axle drag factor is that of the shoulder, \( f_f = 0.45 \) to 0.65, and the rear axle drag factor is the average of the roadway and shoulder, \( f_r = 0.60 \) to 0.78. This difference requires computation for the resultant drag factor using equation (6). The resulting drag factor range is \( f = 0.50 \) to 0.69.

<table>
<thead>
<tr>
<th>Section</th>
<th>Distance (ft)</th>
<th>Drag factor</th>
<th>Initial velocity (ft/sec)</th>
<th>End velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E to F</td>
<td>64</td>
<td>0.45 - 0.65</td>
<td>43.1 - 51.8</td>
<td>0.00 - 0.00</td>
</tr>
<tr>
<td>D to E</td>
<td>10</td>
<td>0.50 - 0.69</td>
<td>46.6 - 55.9</td>
<td>43.1 - 51.3</td>
</tr>
<tr>
<td>C to D</td>
<td>81</td>
<td>0.60 - 0.78</td>
<td>72.8 - 84.8</td>
<td>46.6 - 55.9</td>
</tr>
<tr>
<td>B to C</td>
<td>10</td>
<td>0.64 - 0.81</td>
<td>75.6 - 87.8</td>
<td>72.8 - 84.8</td>
</tr>
<tr>
<td>A to B</td>
<td>21</td>
<td>0.75 - 0.90</td>
<td>82.0 - 94.5</td>
<td>75.6 - 87.8</td>
</tr>
<tr>
<td>Overall</td>
<td>186</td>
<td>0.56 - 0.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 23. Summary of example of vehicle skidding over different surfaces.
Finally, the section drag factors are entered into Exhibit 23. Minimum and maximum values for the initial and end velocities for each section are then calculated and inserted into Exhibit 23 to come up with the initial velocity when skidding started. The resulting initial velocity range is 82.0 to 94.5 ft/sec. Converted to mi/hr, the range is 56 to 64 mi/hr.

For this example, the estimate can be simplified without much loss of accuracy by neglecting the two short transitional sections B to C and D to E. The first section is then entirely on the roadway, the second section half on the roadway and half on the shoulder, and the third entirely on the shoulder. The vehicle drag factor for the second section is the average of those provided by the shoulder and the roadway. The lengths of these sections are determined by the point at which the center of mass crosses the edge of the roadway, making the data as follows:

<table>
<thead>
<tr>
<th>Section</th>
<th>Length</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE to F</td>
<td>69</td>
<td>0.45</td>
<td>0.65</td>
</tr>
<tr>
<td>BC to DE</td>
<td>91</td>
<td>0.60</td>
<td>0.78</td>
</tr>
<tr>
<td>A to BC</td>
<td>26 ft</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>All</td>
<td>186 ft</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using these data and the same equations as before, the resulting initial speed range for the entire 186 ft is 56 to 65 mi/hr, compared to 56 to 64 mi/hr for the more accurate calculations. Therefore, no significant difference is found between the two methods.

9. DRAG FACTORS AFTER A COLLISION

Perhaps the most difficult situation for estimating an appropriate drag factor is after a collision. This is complicated by several conditions:

- One or more wheels may be jammed in the damage and are unable to rotate or have their rotation restricted.
- The vehicle will most likely have rotated, causing the weight to shift from one set of axles to the other and from one side to the other.
- After the collision, it is possible that the vehicle will have traveled over more than one surface.
- In the case of a two-vehicle collision, the vehicles may stay together, which can produce complicated after-collision motion.
- The vehicles' shapes are often considerably altered (even to the extent of a vehicle being in two pieces), which may cause tires and other parts of the vehicle to be in contact with the surface.

It is often necessary to know deceleration rates (drag factors) after a collision, because this is needed to do a momentum analysis (see Topic 868 for a discussion of momentum considerations).

**Two Vehicles Engaged After Collision**

After a two-vehicle collision, the vehicles sometimes move off as one unit. This may occur as shown in Exhibit 24. The bullet car front has struck the target car's side. Because the target car had little forward movement at the time of the collision and the principal direction of force (thrust) was directed through each vehicle's center of mass, the two cars stayed engaged and moved together, essentially in a straight line, to their rest position. The question to be answered is the velocity of the two vehicles as they moved off after the collision.

The data given for this example are as follows:

- Target car weight equals 2,200 lbs.
- Bullet car weight equals 3,100 lbs.
- After-collision travel distance equals 135 ft.

![Exhibit 24](image-url). Because the 2,200-lb. car has essentially zero velocity and the direction of force in the collision is through both vehicles' centers of mass, the vehicles move down the road together in the same direction in which the striking car was traveling before the collision.
• Pavement coefficient of friction ($\mu$) equals 0.80.
• The road is level.

For an actual case like this, you should inspect the bullet car to see whether the front wheels are jammed in as a result of the collision. It is also possible that as a result of the collision, the front wheels of the bullet car are partially supported by the target car. This, of course, adds more weight to the target car. For this case, assume that the front wheels of the bullet car were jammed due to damage and would not rotate.

If equal weight is assumed for each car's wheels as they moved to their final position, a simple estimate of the drag factor of the unit (the two cars together) can be easily calculated. Recall that drag factor is simply the total force in the direction of the acceleration divided by the vehicle's weight ($f = F/w$). The weight on each wheel of the target car would be 550 lbs ($2,200/4 = 550$). The weight on each wheel of the bullet car would be 775 lbs ($3,100/4 = 775$). All four wheels on the target car would have been sliding because the car was traveling completely sideways. Thus, the drag factor on each wheel of the target car would be equal to the coefficient of friction. The bullet car would have drag factors of the front wheel equal to the coefficient of friction. However, there is no reason to believe that the rear wheels would be locked due to damage. For this case, assume that the bullet car is a front-engine, front-wheel-drive car. Thus, the drag factor on the rear wheels of the bullet car is equal to the rolling resistance ($f = 0.01$).

The drag force ($F$) in pounds on each wheel can be calculated by multiplying the drag factor ($f$) of each wheel times the weight ($w$) in pounds on the wheel ($F = fw$). The total drag force on the unit (both cars together) is the sum of the drag forces on each wheel. Then the drag factor for the total unit is equal to the total drag force divided by the total weight of the unit. The drag factors and forces for each wheel are now entered in Exhibit 25.

In virtually all cases, the weight on each wheel of a car will be different, even in a static condition. In a dynamic condition as the vehicles decelerate to a stop, it is even more likely that the weights will be different. For the case depicted in Exhibit 24, it is more important to evaluate the effect of the increased load on the front axle (and of course the reduced load on the rear axle) of the bullet car. This can be done as before by developing another table like Exhibit 25.

Assume that 70 percent of the bullet car's weight is on the front axle. Then the weight on the front axle will be 2,170 lbs and the rear axle will be 930 lbs. Assume the weight is equal on each of the two front wheels (1,085 lbs) and on the rear wheels (465 lbs). As long as the four wheels on the target car are sliding, the distribution of the target car's weight over the four wheels will not affect the final drag factor for the unit (two cars together). Insert these values into Exhibit 26.

![Exhibit 26](image)

Exhibit 26. Calculations to determine the drag factor for the after-collision movement of the combined vehicles shown in Exhibit 24 with 70 percent of the bullet car's weight on the front axle.

As before, the total drag force is divided by the total weight of the unit to calculate the unit's drag factor. A drag factor of 0.66 is obtained (3,510/5,300). When equal weight on all wheels was assumed for the bullet car, the drag factor was 0.57; this time it is 0.66. When the initial speed after the collision is calculated, there is not as much difference as you might think. The calculations are shown below.

For drag factor of 0.57:

\[
V_i = \sqrt{\frac{V_e^2 - 2ad}{2}}
\]

\[
= \sqrt{-2(-0.57)(32.2)(135)}
\]

\[
= 70.4 \text{ ft/sec}
\]

\[
= 47.9 \text{ mi/hr}
\]

For drag factor of 0.66:

\[
V_i = \sqrt{\frac{V_e^2 - 2ad}{2}}
\]

\[
= \sqrt{-2(-0.66)(32.2)(135)}
\]

\[
= 75.7 \text{ ft/sec}
\]

\[
= 51.5 \text{ mi/hr}
\]

Clearly, there is not a lot of difference in mi/hr for the two sets of calculations. This example could be worked another:

62-29
time with even more weight shifted to the front axle of the bullet car. The equation for weight shift could be used as before for braking. However, the car is obviously modified somewhat because of damage, which may make the assumptions for horizontal and vertical locations of the center of mass incorrect if you have assumed the characteristics of an undamaged car.

**Single Vehicle Translating and Rotating After Collision**

Consider the following situation. A car has significant front-end damage. The damage is completely across the front of the car, and the front wheels are jammed-in due to the damage. Neither front wheel will rotate because of the damage. The car rotates counterclockwise 180 degrees, while at the same time it translates a distance, d, until it comes to rest. This situation is shown in Exhibit 27.

At position A the car is essentially moving sideways; not only are the front wheels not rotating (they are locked because of damage) but also the rear wheels are not rotating because of the sideways movement. Therefore, at position A the drag factor on the vehicle would be equal to the coefficient of friction. At position B the rear wheels are free to roll, while the front wheels provide a drag factor of more than 50 percent of the friction coefficient (if more than 50 percent of the vehicle's weight is on the front wheels). At position C the vehicle is again moving sideways, so the drag factor is equal to the coefficient of friction. Thus, it can be seen that the drag factor at position A decreases to something near one-half the coefficient of friction at position B and then increases to the maximum again at position C. This is plotted in Exhibit 28 as a function of distance.

If the vehicle in Exhibit 27 decelerated to a stop at position C, the initial velocity at position A could be calculated. Most likely, you would want to use the following equation:

\[ v_i = \sqrt{v_e^2 - 2ad} \]  

(1)

However, acceleration (a) or drag factor (f) constantly changes from positions A to B to C. Thus, to use the above equation, an average value of drag factor would have to be assumed for the entire distance. An option that should yield more accurate results is to divide the total distance into smaller distances and apply the equation for each of the smaller distances. For each distance increment a different drag factor would be used. For example, note how the distances are divided into equal increments in Exhibit 29. Each increment has a drag factor associated with it. (That is, for \( d_1 \) use \( f_1 \), for \( d_2 \) use \( f_2 \), etc.). You would then start at the rest position, C, and “back up”, using the above equation until you get to position A.

Another option to determine the initial velocity at position A is to calculate the work done in each increment of distance shown in Exhibit 29. The work done is simply the drag factor times the distance times the weight. The total work is simply the sum of the work done in each distance increment. Equate this to the initial kinetic energy at position A and solve for the initial velocity. In equation form this becomes:

\[ Work_{total} = f_1wd_1 + f_2wd_2 + f_3wd_3 + f_4wd_4 + \ldots \]  

(7)

\[ v = \sqrt{\frac{2g \text{work}_{total}}{m}} \]  

(8)

The number of terms for the right side of equation (7) depends on the number of surfaces. You could have more than four terms, as indicated in equation (7).

Often a vehicle does not travel over the same surface after a collision. If, for example, the car that rotated 180 degrees in Exhibit 27 traveled over three surfaces, the coefficient of friction would most likely be different over each surface. Thus, the resulting drag factors on the car as it travels to a stop would also be different. If it is known how the vehicle moves over the surface (that is, how it translates and rotates), the same approach can be used. Say, for example, that the car in Exhibit 27 first traveled over a hard surface for the first third of its total after-collision distance, then traveled for the next third over gravel, and for the final third over grass. Test values for the coefficients of friction were found to be 0.80 for pavement, 0.50 for gravel, and 0.40 for grass. Exhibit 30 shows how the drag factor could be re-plotted to show how drag factor varied on the vehicle from position A to B to C.

**Case Study: Front Half of Car**

Exhibit 31 shows the front part of a car. The car hit a large utility pole broadside and broke into two pieces. One question that needs to be answered is the speed of the front part of the car after it hit the pole. So the problem is to determine the drag factor of the “vehicle” (that is, the front of the car) as it traveled approximately 80 ft after the collision.

There were no marks in the grass to indicate that the vehicle was digging into the turf as it moved to its final position. There is no reason to believe that the vehicle was moving through the air. No apparent vertical component of force was acting on the vehicle to make it travel through the air. Thus, the vehicle must have been in contact with the ground. An eccentric force was applied to the vehicle that would have caused it to rotate at least 180 degrees. Inspection of the vehicle showed that the two wheels were free to rotate. Actually, the two wheels looked to be essentially normal. This is a rear-wheel-drive car, so there was no transaxle to act as a decelerating force on the two wheels. Therefore, it would appear that a two-wheeled vehicle (almost like an ox cart) rotated at least 180 degrees and translated with free-rolling wheels approximately 80 ft without digging into the turf. This does not suggest a very high drag factor. Clearly, it would be more than rolling resistance on a hard surface. The grass would be more difficult to roll over than would a hard, smooth surface. The vehicle did rotate. However, consider the effort required to
Exhibit 27. The car shown in the drawing has its front wheels locked due to damage. At A it is sliding essentially sideways while it continues rotating counter-clockwise. The deceleration on the car decreases as it moves to A. At B the rear wheels are free to roll. Then, as the car moves to C, the deceleration on the car continues to increase until the drag factor is equal to the coefficient of friction.

Exhibit 28. The drag factor has been plotted against distance for the movement of the car shown in Exhibit 27. Notice how the drag factor decreases from A to B and increases from B to C.

Exhibit 29. To calculate the velocity of the center of mass of the vehicle at A shown in Exhibit 27, you can divide the total distance into smaller increments and use an average drag factor for each distance.

Exhibit 30. If the vehicle shown in Exhibit 27 traveled over several surfaces from A to C, you must take into account the difference in the drag factor caused by this. One way is to determine how the drag factor varied over each surface, as illustrated here.
Exhibit 32. This photograph shows two cars still engaged in their rest positions. In this case, the cars clearly attain a common velocity.

Exhibit 31. The front part of this car was separated from the rest by a side impact with a pole. The wheels were free to roll as it moved to its final position after separating from the pole. If no part of the car front dug into the ground, it would be difficult to support a high after-collision drag factor.
rotate a two-wheeled trailer. If the weight is balanced fairly evenly over the two wheels, one person can push the tongue sideways and cause it to rotate without any great effort. If rolling resistance (drag factor) is 0.01 on a hard, level surface, then it would be very difficult to support any drag factor in excess of 0.10 for this case.

A velocity estimate of the vehicle as it left the pole can be done by using the equation \( a = f g \) and equation (1):

\[
\begin{align*}
\nu_i &= \sqrt{\nu_e^2 - 2ad} \\
&= \sqrt{0^2 - (2)(-0.10)(32.2)(80)} \\
&= 22.7 \text{ ft/sec} = 15 \text{ mi/hr}
\end{align*}
\]

An argument could be made that the drag factor is too high. However, if this speed estimate is combined with the speed required to break the car into two pieces, it can be clearly seen that this is a fairly insignificant value. With the data given, it would be very difficult to support a larger drag factor. For example, if someone suggests a value approaching 0.50, there would be nothing in the data to support such a conclusion.

**Case Study: Two Vehicles Together from Maximum Engagement to Rest Positions**

Exhibit 32 shows two vehicles (Plymouth and Pontiac) still together at their rest positions. Exhibit 33 shows the after-accident situation map. Exhibit 34 shows maximum-engagement positions of the two vehicles and intermediate positions as the vehicles travel together to their rest positions. Test skids on the Portland cement concrete gave a coefficient of friction of 0.75. Test skids on the grass gave a coefficient of friction of 0.45. The data clearly supports that the Pontiac slid sideways the entire distance after the collision. Thus, all four of its wheels were sliding without rotation. As a result of the collision, the Plymouth's driveshaft came off the car (note its final position). Thus, the rear wheels were free to rotate. This case is similar to the scenario described in Exhibit 24.

As a first-cut analysis, assume that the weights on all the wheels were equal when the two vehicles moved as one unit to their rest positions. If that was the case, then six of the eight wheels were not rotating, while the other two were free to rotate. If the weight was equal on all eight wheels, then the drag factor acting on the combined unit would be six-eighths, or 75 percent of the drag factor over the two surfaces.

Thus, the drag factors were:

Grass: \( f = 0.75 \mu \)
\[
= (0.75)(0.45)
= 0.34
\]

Concrete: \( f = 0.75 \mu \)
\[
= (0.75)(0.75)
= 0.56
\]

The cars moved as a unit for 40 ft over grass and 80 ft over the concrete. Then the velocity after maximum engagement can be calculated using equation (1) twice, as follows:

\[
\begin{align*}
v_i &= \sqrt{\nu_e^2 - 2ad} \\
&= \sqrt{0^2 - 2(0.34)(32.2)(40)} \\
&= 29.6 \text{ ft/sec}
\end{align*}
\]

Thus, as the cars start to slide on the grass, they are traveling 29.6 ft/sec. Use this value as the end velocity, \( v_e \), in equation (1):

\[
\begin{align*}
v_i &= \sqrt{\nu_e^2 - 2ad} \\
&= \sqrt{29.6^2 - 2(-0.56)(32.2)(80)} \\
&= 61.3 \text{ ft/sec}
\end{align*}
\]

The approach used above assumes that the weight on each wheel is equal. Clearly, this is not going to be the usual situation. If the rear axle of the bullet car (Plymouth) had less than the assumed weight, then the assumed drag factor for the two vehicles together after the collision would be less than what the two vehicles would experience. To check the sensitivity of this analysis, assume that the rear axle has 20 percent of the Plymouth's weight. The gross weight of the Plymouth is 3,700 lbs, that of the Pontiac 3,600 lbs. The easiest way to understand the effects of the assumptions is to create a table similar to Exhibit 26. This has been done in Exhibit 35 for the sliding across the pavement. The target car (Pontiac) has been assumed to have equal weight on all four wheels. This clearly is not the case. However, because all four wheels were sliding sideways, there will be no difference in the total resisting force. If you had a case where the weight distribution was not equal and all the wheels were not sliding, this should be considered. The total resisting force is 4,930 lbs and the total weight is 7,300 lbs (3,600 + 3,700). Thus, the drag factor across the pavement is 4,930/7,300 (\( f = F/w \))

<table>
<thead>
<tr>
<th>Wheel</th>
<th>Plymouth</th>
<th>Pontiac</th>
<th>Total F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w f F</td>
<td>w f F</td>
<td></td>
</tr>
<tr>
<td>RF</td>
<td>1,480 0.75 1,110</td>
<td>900 0.75 675</td>
<td>1,785</td>
</tr>
<tr>
<td>LF</td>
<td>1,480 0.75 1,110</td>
<td>900 0.75 675</td>
<td>1,785</td>
</tr>
<tr>
<td>RR</td>
<td>370 0.01 5</td>
<td>900 0.75 675</td>
<td>680</td>
</tr>
<tr>
<td>LR</td>
<td>370 0.01 5</td>
<td>900 0.75 675</td>
<td>680</td>
</tr>
</tbody>
</table>

**Exhibit 35.** Calculations to determine the drag factor on the Plymouth/Pontiac vehicle as it moves across the pavement, with 20 percent of the Plymouth weight on the rear axle and an equal weight distribution on the Pontiac wheels.
Exhibit 33. This diagram shows the after-accident situation map for the cars in Exhibit 32.

Exhibit 34. The movement of the two vehicles from engagement to final positions is depicted here.
or 0.68. The first estimate of the pavement drag factor (assuming equal weight on all eight wheels) was 0.56.

Another table can be developed for the sliding across the grass. This is done in Exhibit 36. Again, an equal weight distribution is assumed for the wheels of the Pontiac. The total sliding force is 2,960 lbs. This gives a drag factor of 2,960 / 7,300, or 0.41. The previous drag calculation for equal weight on all wheels was 0.34.

<table>
<thead>
<tr>
<th>Wheel</th>
<th>Plymouth</th>
<th>Pontiac</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w</td>
<td>f</td>
</tr>
<tr>
<td>RF</td>
<td>1,480</td>
<td>0.45</td>
</tr>
<tr>
<td>LF</td>
<td>1,480</td>
<td>0.45</td>
</tr>
<tr>
<td>RR</td>
<td>370</td>
<td>0.01</td>
</tr>
<tr>
<td>LR</td>
<td>370</td>
<td>0.01</td>
</tr>
<tr>
<td>Total Drag Force</td>
<td>2,960 lbs</td>
<td></td>
</tr>
</tbody>
</table>

Exhibit 36. Calculations to determine the drag factor on the Pontiac/Plymouth vehicle as it moves across the grass, assuming 20 percent of the Plymouth's weight is on the rear wheels.

Using the drag factor of 0.41 for the grass and 0.68 for the pavement, let's see what difference it makes. Again using Equation (1):

\[ v_i = \sqrt{v_e^2 - 2ad} \]  \hspace{1cm} (1)

\[ = \sqrt{0^2 - 2(-0.41)(32.2)(40)} \]

\[ = 32.5 \text{ ft/sec} \]

Thus, as the cars start to slide on the grass, they are traveling 32.5 ft/sec. Use this value as the end velocity, \( v_e \), in Equation (1):

\[ v_i = \sqrt{v_e^2 - 2ad} \]  \hspace{1cm} (1)

\[ = \sqrt{32.5^2 - 2(-0.68)(32.2)(80)} \]

\[ = 67.5 \text{ ft/sec} \]

The previous estimate was 61.3 ft/sec. Thus, the difference is 6.2 ft/sec or 4.2 mi/hr. In this case, very little change resulted from using the “rough cut” method — mainly because the vehicle weights were very similar, which helped to minimize the difference.

The after-collision speeds calculated for the Pontiac and Plymouth are not the speeds of either car at first contact. To calculate those speeds, a momentum analysis must be done. This is discussed in Topic 868.

**10. VEHICLE CENTER OF MASS LOCATION**

Earlier in this topic the height and horizontal location of a vehicle's center of mass were used in a calculation to determine the resultant drag factor. Published data on vehicle dimensions rarely include location of center of mass. However, manufacturers publish information giving the amount of weight on front and rear axles separately or the fraction (percent) of weight on one axle, which amounts to the same thing. From this available information, the longitudinal (fore and aft) position of the center of mass can be calculated, but not its height.

**Data needed** to calculate the location of the center of mass are as follows:
1. Static weight on the front \( (w_f) \) and rear \( (w_r) \) axles or the fraction (percent) of weight on either front or rear axle;
2. Wheelbase of vehicle \( (l) \) — that is, the distance from the front to the rear axle;
3. Radius of wheels with tires \( (r) \);
4. Weight on one axle \( (w_h) \) when the other axle is lifted to a height \( (h) \);
5. Height \( (h) \) to which the axle is lifted for weighing to locate the center of mass.

Symbols for these quantities are given in Exhibit 37. Measurements can be made to obtain values for these quantities with a particular vehicle.

**Center of Mass Longitudinal Position**

The longitudinal (fore and aft) position of the center of mass requires knowledge of the static (standing) weight on the front and rear axles and wheelbase length (the first two items in the list above).

The most useful way to describe the longitudinal position of the center of mass is as a decimal fraction of the wheelbase behind the front axle, \( x_l \). The quantity \( x_l \) is equal to \( l_t/l \) (see Exhibit 37 for \( l_t \)). Therefore, the actual distance of the center of mass to the rear of the front axle is calculated as follows:

\[ l_t = x_l l \]  \hspace{1cm} (9)
In terms of static weight on the front axle,

\[ x_f = w_f / w \]  

(10)

where \( w \) is equal to the total weight of the vehicle. Similar equations can be developed to obtain a horizontal center-of-mass measurement from the rear axle. The wheelbase length (expressed as a decimal fraction) between the center of mass and one axle is proportional to the vehicle weight on the other axle. If a vehicle's weight distribution is given as a percentage of total weight on one axle, the vehicle's center of mass is the same percentage of wheelbase length away from the other axle.

To obtain the static weight on front and rear axles, drive only the front wheels of the vehicle on a commercial platform scale and weigh it. Then place only the rear wheels on the scale and weigh again. The entire vehicle may be weighed to verify the measurements. Therefore, \( w_f + w_r = w \).

Example. On a given car, the front wheels (axle) carry a weight, \( w_f \), of 1,750 lbs, and the rear a weight, \( w_r \), of 1,425 lbs. The total weight of the vehicle is therefore 1,750 + 1,425 = 3,175 lbs. The position of the center of mass behind the front axle is

\[ x_f = \frac{w_r}{w} = \frac{1,425}{3,175} = 0.449 \]

The wheelbase of this car measures 9.39 ft. The actual distance of the center of mass behind the front axles is therefore

\[ L_f = x_f L = (0.449)(9.39) = 4.22 \text{ ft} \]

Center of Mass Height

Additional data for the height of the center of mass above the ground are obtained by weighing the front or rear axle and wheels on a platform scale when the other axle (not just the body) is lifted several feet off the ground.

A tow truck is usually necessary to lift the axle high enough, about a third of the wheelbase of the vehicle. For a full-size car, this is 3 ft (1m) or more. If the hoist is less, the change in weight will be too small to be recorded adequately on a commercial platform scale, which usually weighs only in increments of 25 lbs (10kg). Note that the amount of hoist, \( h \), is the distance the axle is lifted, not its height above the ground (that would be \( h + r \)). The quantity \( r \) is the radius of the wheel with tire. The distance of the tire from the ground is the same as the axle lift and can often be more easily measured. With the required data at hand, the equation of the height of the center of mass is, with rear end lifted as in Exhibit 37.

\[ l_z = \frac{l \sqrt{l^2 - h^2 (w_h - w_r)}}{h w} + r \]

(11)

where \( l \) = wheelbase in ft or m,

\[ l_z = \text{height of the center of mass in ft or m}, \]

\[ h = \text{the height the rear wheels were hoisted in ft or m}, \]

\[ r = \text{radius of tire and wheel together in ft or m}, \]

\[ w_f = \text{level weight on front axle in lbs or N}, \]

\[ w_r = \text{weight on front axle after hoisting in lbs or N} \]

If the front of the car is lifted, the term \((w_h - w_r)\) is replaced by \((w_h - w_f)\). Of course, \( w_h \) would be the weight on the rear after it was lifted, and \( w_f \) would be the level static weight on the rear wheels.

The height of the center of mass, expressed as a decimal fraction of the wheelbase, is

\[ z = l_z / l \]

(12)

Example. The vehicle is the same as in the previous example (see Exhibit 38 for the following dimensions). The wheelbase, \( l = 9.39 \text{ ft} \); weight on the rear axle with the car level, \( w_r = 1,425 \text{ lbs} \); weight on rear axle with front axle hoisted \( h \) ft, \( w_h = 1,525 \text{ lbs} \); total weight, \( w = 3,175 \text{ lbs} \); distance from ground to wheel center with wheel hoisted, 3.43 ft; radius of wheel with tire, \( r = 0.99 \text{ ft} \). Then the distance the axle was hoisted is \( h = 3.43 - 0.99 = 2.44 \text{ ft} \). Substitute these values in the equation for \( l_z \) with the front lifted:

\[ l_z = \frac{9.39 \sqrt{9.39^2 - 2.44^2} (1,525 - 1,425)}{2.09(3,175)} + 0.99 \]

\[ l_z = 8.514 / 7747 + 0.99 \]

\[ l_z = 1.00 + 0.99 \]

\[ l_z = 2.09 \text{ ft} \]

Expressing this as a decimal fraction of the wheelbase, you obtain

\[ z = l_z / l = 2.09 / 9.39 = 0.226 \]

Accuracy. The greatest source of error is in weighing the vehicle. In the example, the weight of the rear wheel with vehicle level is 1,425 lbs. But the scales weigh only to the nearest 25 lbs. Therefore, the actual weight might be as much as 1437.5 or as little as 1412.5 lbs. The weight on the rear wheel with the front wheel hoisted might be as much as 1537.5 and as little as 1512.5 lbs. The difference of these weights, \( w_h - w_r \), might be as little as 75 or as much as 125 lbs. Using this range of values in the equation with the measured amount of hoist gives a possible range of values for \( z \) of 0.198 to 0.252, and a possible range of 1.81 to 2.45 ft for the actual height of the exemplar car's center of mass. That would be a spread of 0.65 ft, or about 7.8 in. (20.6 cm).
The height of the center of mass can also be found by weighing one side with the other side hoisted. The same equation is used, except that the track width of the vehicle is used instead of the wheelbase. Hoisting one side of the vehicle, however, is less convenient than hoisting one end, especially given the layout of most available platform scales.

If data are not available to calculate the height of the center of mass, a useful estimate can be made of its position. The center of mass is generally near where the most weight is concentrated. Hence, except in unusual vehicles, it would not be below the center of the wheels or much above the top of the wheels. By noting whether the engine and other heavy parts are set high or low, a reasonable judgment can usually be reached as to a range of levels between which the height of the center of mass would be located. The limit of this range can be used in preliminary calculations, which will indicate whether more refined methods of locating the height of the center of are needed. If calculations made using the estimated high and low figures do not differ significantly, some estimated figure between them can be used with reasonable confidence that calculated results are not greatly in error.

Crossway Location of Center of Mass

Crossway position of center of mass can also be found by weighing right and left wheels separately and using track width instead of wheelbase length. In most vehicles, the center of mass is very close to the longitudinal (lengthwise) centerline of the vehicle. Therefore, unless there is reason to believe that there is a significant difference between load on left and right wheels, one can assume that the center of mass is on the longitudinal centerline. The right and left positions of the center of mass have little significance in most problems relating to slowing to a stop.

Load Effects

Load must be included in center-of-mass calculations. If the vehicle is available as loaded, it is not difficult to determine the longitudinal position of the center of mass by weighing the front and rear wheels separately. If the load has been removed from the vehicle or if the vehicle and load are no longer available, it is often possible to obtain a similar vehicle and load it, for purposes of testing, approximately as the actual vehicle was loaded. However, it is usually impractical to tilt a loaded vehicle to obtain the height of the center of mass. Then the vehicle's center of mass and the load's center of mass may be estimated separately and combined by calculations. Sample calculations illustrating this methodology are shown in the topic on heavy trucks (Topic 878).

11. OTHER ACCELERATION VALUES

As stated earlier in this topic, the quantity $f$ is simply the number of g's of acceleration a vehicle has. Normally, $f$ is used in the context of slowing (that is, decreasing velocity). The quantity $f$ can also be used for positive acceleration (increasing velocity). If a car pulls away after stopping at a stop sign, the car is clearly accelerated. It may be useful to calculate the maximum velocity a car could have accelerated by using an appropriate acceleration rate. Typical values (and the resulting acceleration values in ft/sec$^2$ and m/sec$^2$) are shown in Exhibit 39. Also shown in Exhibit 39 are typical deceleration values for vehicles that do not have locked wheel skids. Note that positive (+) and negative (−) signs are used to indicate positive acceleration (increasing velocity) and negative acceleration (decreasing velocity) conditions.

Exhibit 39 gives typical values for normal and rapid acceleration rates of passenger cars. As would be expected, a car cannot maintain the same acceleration rate as its speed increases. Representative values are suggested in Exhibit 39 to reflect this. Some cars may have better performance values than the values listed. Nevertheless, the driver has the ultimate decision on how much acceleration is selected.

Typical slowing drag factors are included in Exhibit 39. As noted, these are not skidding values. They have been included to give you an idea of typical deceleration values encountered under several situations that do not involve locked wheel skids or very hard braking.

12. SUMMARY

This topic has explained the difference between drag factor and coefficient of friction. Clearly, they are not the same quantities. Perhaps the most often observed misconception regarding drag factor occurs when after-collision drag factors are estimated. A drag factor is often assumed to be equal to the coefficient of friction for a vehicle after a collision. Usually this is not the case. Such an assumption can result in mistakenly high values for first-contact speeds.

Other errors in estimating drag factors stem from assuming that large trucks and motorcycles have the same values cars have. Rarely is this appropriate. Large-truck and motorcycle drag factors are addressed in Topics 874 and 878. Consult these for more information.

Determining drag factors for automobiles and pickup trucks is not always an easy task. Test skids of exemplar vehicles can help considerably. If tests are not done, tables
such as Exhibit 15 can provide a usable range of values.

If you are using an equation to calculate speed from braking skids, be certain that you indeed have braking skids and not some other type of tire mark. Failure to identify the correct type of tire mark will lead you to use inappropriate equations for speed estimates, even if you have done test skids. Review the topic on tire marks (Topic 817) if you are not sure what you have.

13. SOURCES

Authors

Lynn B. Fricke is a traffic engineer specializing in accident reconstruction. He has been with The Traffic Institute since 1975. In 1981 he became the director of the Institute's Accident Investigation Division.

J. Stannard Baker is a traffic engineer specializing in accident reconstruction. He was director of research and development at The Traffic Institute from 1946 to 1971 and is a guest lecturer for the Institute.

References

Superscript numbers in the preceding pages refer to the following publications:


Exhibits

The following are the sources of the exhibits used in this topic.

Baker, J., Stannard, San Diego, CA
Photos: 13
Diagrams: 18 - 20, 37 - 38
Tables: 15 - 17, 22, 39
Graphs: 3

Baker, Kenneth S., Evanston, IL
Photos: 7 – 8

Beck, Michael, Indiana Law Enforcement Training Academy, Plainfield, IN
Photo: 14

Cooper, Gary W., Wauconda, IL
Photos: 32
Diagrams: 33 – 34

Fricke, Lynn B., Lincolnshire, IL
Photos: 5 – 6, 10 – 12
Diagrams: 1 – 2, 24, 27
Tables: 21, 23, 25 – 26, 35 – 36
Graphs: 28 – 30

Garrott, W.R., Ohio State University, Columbus, OH
Graph: 4

Warner, Charles W., Orem, UT
Diagrams: 9

Unknown
Photo: 31
Most of the material in this volume is being published for the first time. The only exceptions are parts of the "Reconstruction" chapter found in the 1975 edition of The Traffic Accident Investigation Manual (Volume 1) and the Traffic Institute publication "Estimating Vehicle Stopping Time and Distance," which also covered certain material used in this book. Otherwise, the topics found in this volume represent new material.


Copyright 1990 by Northwestern University Traffic Institute

Evanston, Illinois 60204

All rights reserved. No part of this book may be reproduced in any form or by any electronic or mechanical means including photocopying, recording, or by any information storage and retrieval system, except for the purpose of short quotations within book reviews, without permission in writing from the publisher.

ISBN 0-912642-07-6

Library of Congress Catalog Number 90-60196

Printed in the United States of America